Position-Dependent Smoothness-Increasing Accuracy-Conserving Filtering for Discontinuous Galerkin Solutions

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The Discontinuous Galerkin (DG) method has become increasingly popular for approximating the solution to PDEs. This is due to its flexibility in handling non-matching grids and in designing *hp*-refinement strategies. It is possible to extract extra orders of accuracy from DG approximations as they contain 'hidden accuracy'. Normally, the convergence rate is of order $k+1$ (or $k+½$), where k is the highest degree polynomial used in the approximation. However, a Fourier analysis reveals an accurate mode that evolves with an accuracy of order $2k + 1$. Position-dependent smoothness-increasing accuracy-conserving (SIAC) filtering is a promising technique to extract this superconvergence. It can not only improve the order of the numerical solution obtained by a discontinuous Galerkin (DG) method but also increase the smoothness of the field and improve the magnitude of the errors. This was initially established as an accuracy enhancement technique by Cockburn Luskin, Shu, and Süli for linear hyperbolic equations to handle smooth solutions [*Math. Comp., 72 (2003), pp. 577–606*] based on the ideas of Bramble and Schatz [*Math. Comp. , 31(1977)*] and Mock and Lax [*Comm. Pure Appl. Math., 31(1978):423–430*]. By implementing this technique, the quality of the solution can be improved from order $k+1$ to order $2k+1$ in the L^2-norm. Ryan and Shu used these ideas to extend this technique to be able to handle postprocessing near boundaries as well as discontinuities [*Methods Appl. Anal., 10 (2003), pp. 295–307*]. However, this presented difficulties as the resulting error had a stair-stepping effect and the errors themselves were not improved over those of the DG solution unless the mesh was suitably refined. In this talk, we discuss an improved filter for enhancing DG solutions that easily switches between one-sided postprocessing to handle boundaries or discontinuities and symmetric post-processing for smooth regions. We numerically demonstrate that the magnitude of the errors using the modified SIAC filter is roughly the same as that of the errors for the symmetric post-processor itself, regardless of the boundary conditions. Furthermore we also present and theoretical error estimates for the accuracy of this position-dependent SIAC filter. More precisely, we show that, for a certain class of DG approximations for linear hyperbolic equations, the position-dependent post-processor enhances the accuracy in the entire spatial domain from from $O(h^{k+1})$ in the L^2-norm for the DG solution to $O(h^{2k+1})$ in the L^∞-norm for the filtered solution. This is joint work with Liangyue Ji and Paulien van Slingerland.