Polygonal Brain, Conformal Transplant, and Alzheimer's Disease

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Few info from alz.org, NIH, webmd, and yes! wiki sites.

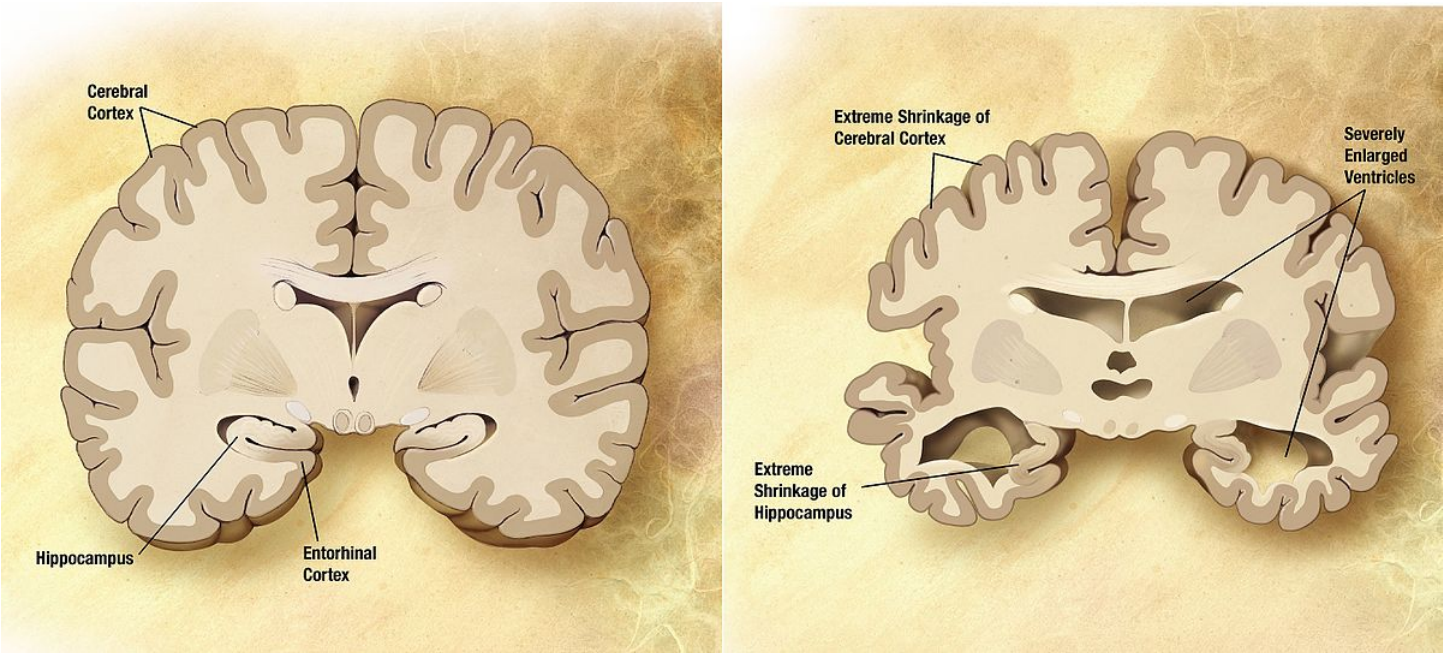
Few facts

- German psychiatrist and pathologist, Alois Alzheimer (1906).
- Mistakenly attributed to part of ageing process.
- Competing hypotheses for causes: Genetics (one of 60 minute shows), amyloid protein plague, etc .
- Stages: initial condition, early, middle, late. Real causes are still not well-understood.

Effects from mild to serious

- Initial stages (not necessarily due to AD): minor short memory loss, try opening office door by pressing car key, forget things occasionally such as teaching load, etc.
- Early: forget names of family members and friends, etc.
- Middle: unable to go back home (or enter the wrong home!). Trouble knowing where you are.
- Late: poor ability to think. Having trouble in doing basic chores.

What is happening to the brain ?



Common Mathematical and Computational Model

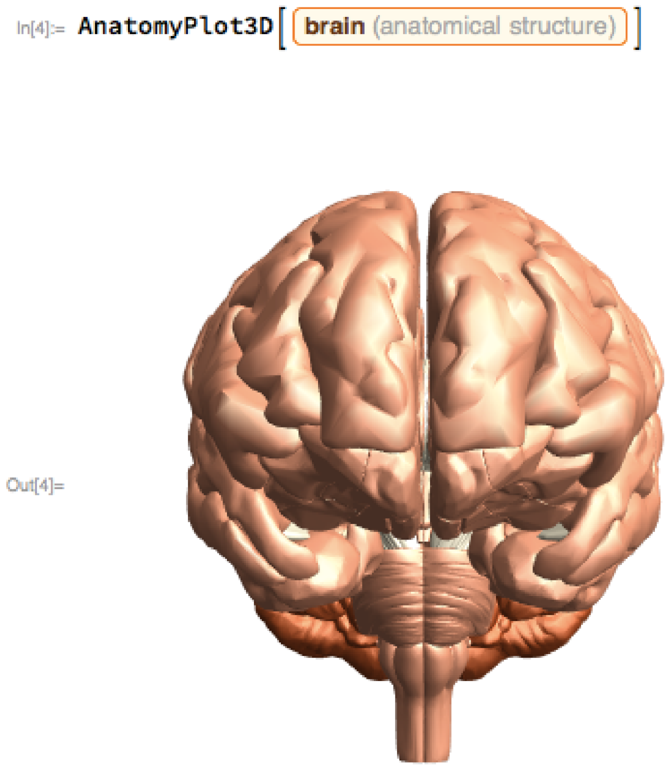
- Predictive modeling: Machine learning based on MRI data.
- Systems of DE (micro model): Chemical reactions for amyloid plague formations.
- Systems of PDE (macro model): Modeling the spread of amyloid plague.

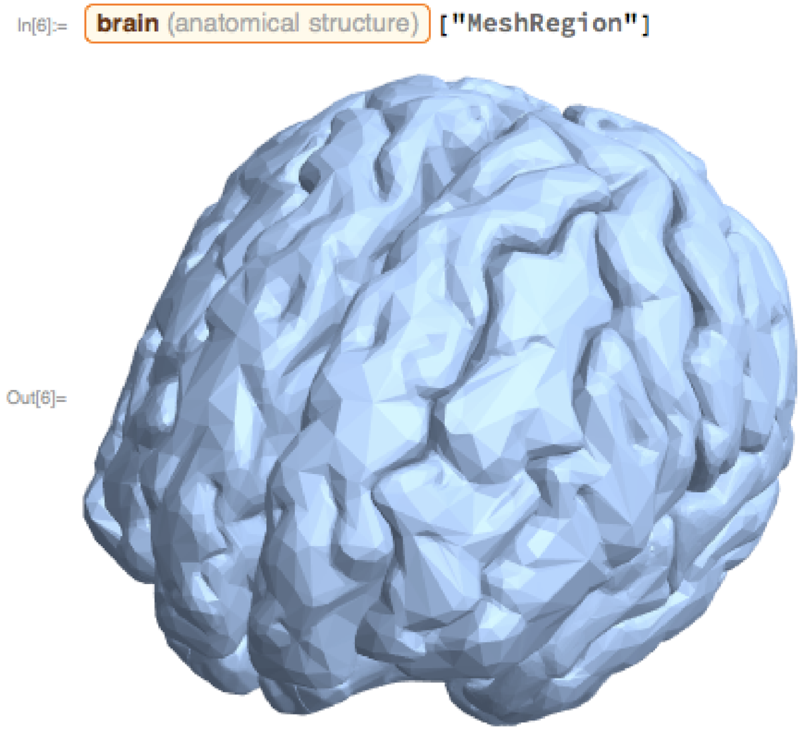
System of PDEs on irregular domain

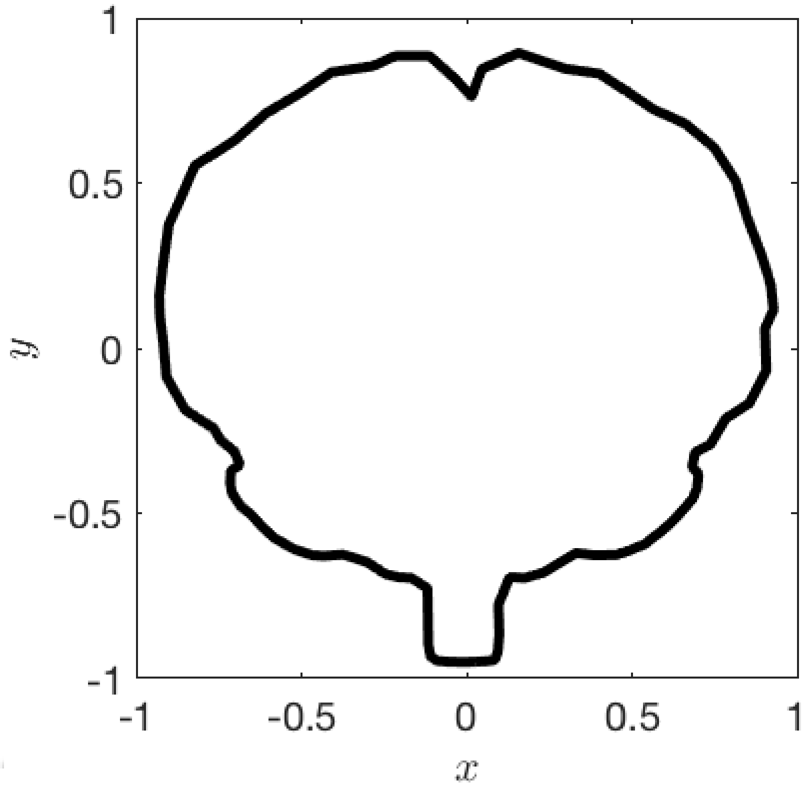
- Systems of reaction-diffusion PDEs [Bertsch, et al. , Hao and Friedman]
- 2D vertical/horizontal cross-section of the brain are geometrically complex (almost like a maze).
- Preliminary goal: simulate this problem with a conformally-mapped pseudospectral method with MATLAB.

Getting the domain of the brain

This is probably for a lazy person like me: Use Mathematica's anatomy 3D. This is also useful if you are a finite-element person.








```
ev = AnatomyData[Entity["AnatomicalStructure","Brain"],"MeshRegion"];
pts = MeshCoordinates[ev];
Export["braincrosssect.mat",pts];
```

Vertical or Horizonal cross-section can be obtained by slicing the brain 3D object with a plane. Take a look at commands: **RegionIntersection** in Mathematica coupled with **boundary** in MATLAB.

System of Reaction-Diffusion Equations on a Unit Disc

Continuous version (e.g. 2 species, simplest version):

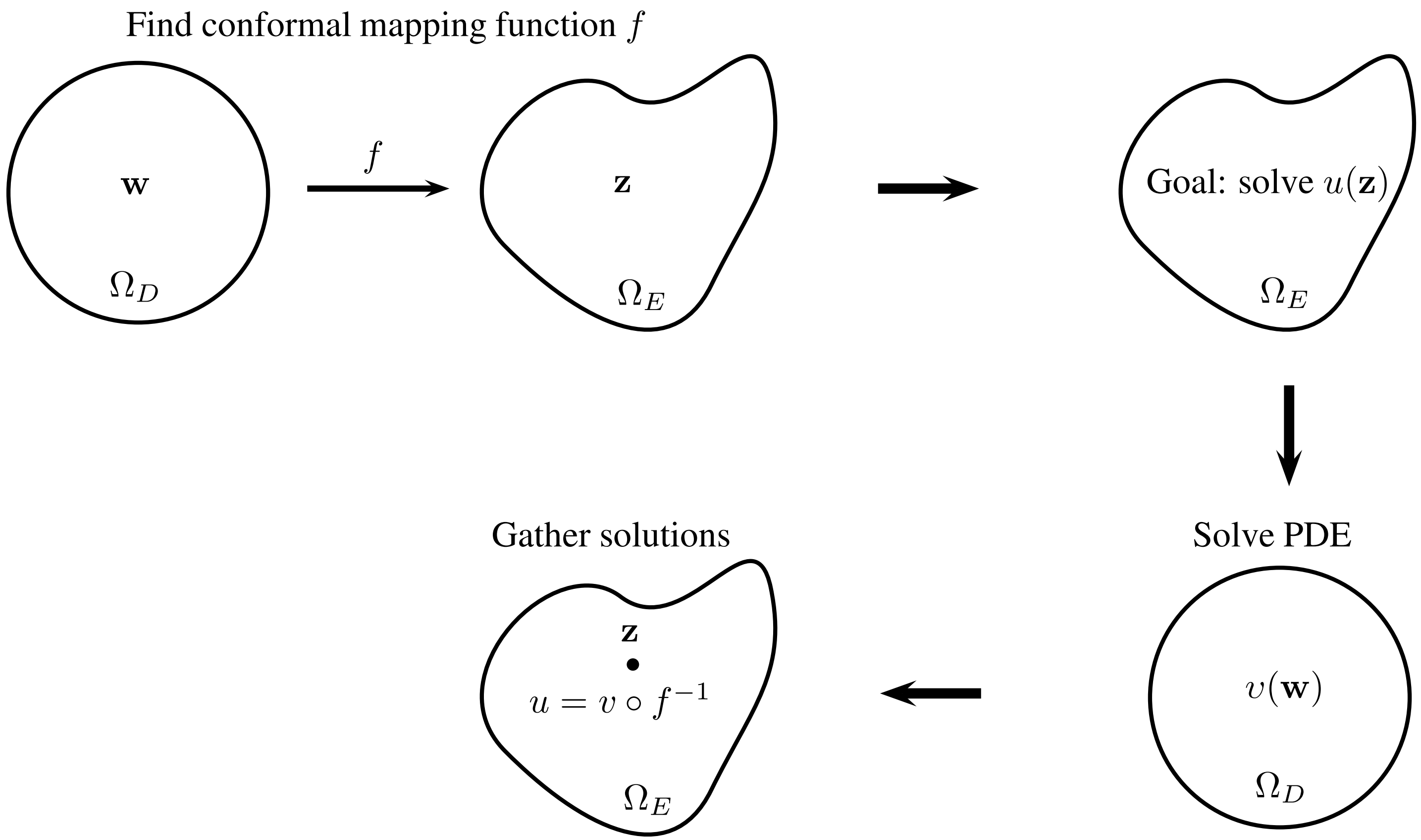
$$
\begin{aligned}
PDE: \quad & \frac{\partial u_1}{\partial t} = d_1(x,y,t)\Delta u_1 + a_{11}(x,y,t)u_1^2 + a_{12}(x,y,t)u_1 u_2 + F_1(x,y,t) \\
& \frac{\partial u_2}{\partial t} = d_2(x,y,t)\Delta u_2 + a_{21}(x,y,t)u_2^2 + a_{22}(x,y,t)u_1 u_2 + F_2(x,y,t) \\
& (x,y) \in \text{unit disc brain} \\
IC: \quad & u_1(x,y,0) = f_1(x,y), \quad u_2(x,y,0) = f_2(x,y) \\
BC: \quad & u_1(x,y) = g_1(x,y), \quad u_2(x,y) = g_2(x,y), \quad \text{or mixed with Neumann.}
\end{aligned}
$$

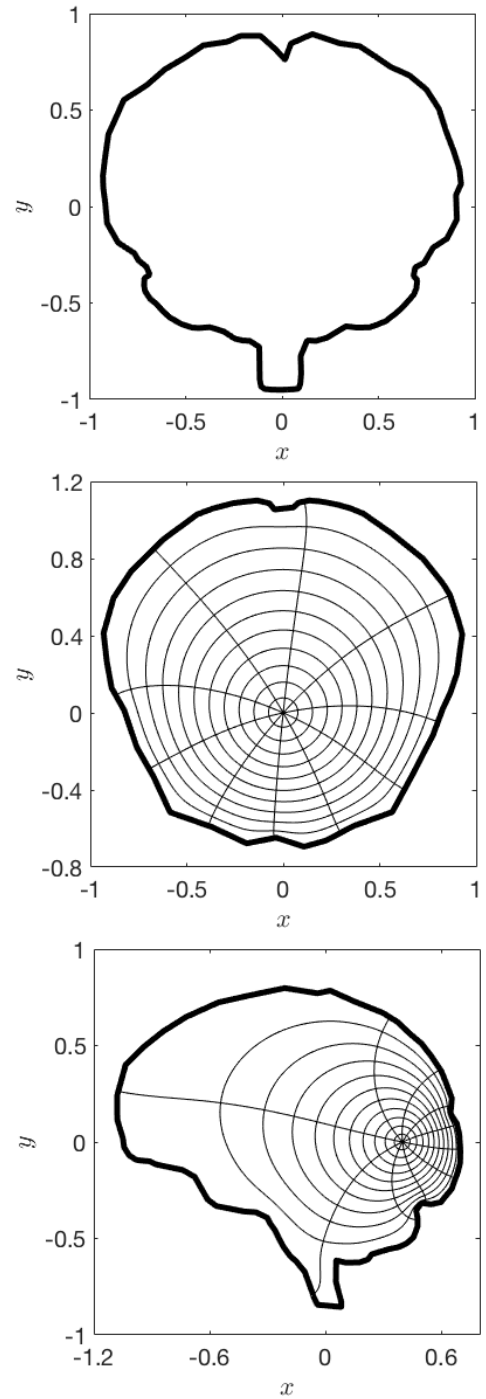
Discrete analogue: march in time with MATLAB standard MOL ode solver.

$$
u_t = \begin{bmatrix} \text{diag}(d_1)\mathcal{L} & 0 \\ 0 & \text{diag}(d_2)\mathcal{L} \end{bmatrix} u + \mathcal{N}(a,u) + \mathcal{F}
$$

ode15s (as ODE or DAE) with adaptive time-stepping: Jacobian is provided.

Conformal Transplantation





Driscoll's Schwarz-Christoffel Mapping Toolbox for MATLAB

`https://github.com/tobydriscoll/sc-toolbox`

(Github, H. added doubly-connected branch)

```
p = polygon(pvert);
options = scmapopt('Tolerance',1e-14);
fmap = diskmap(p,options);
plot(fmap);
```

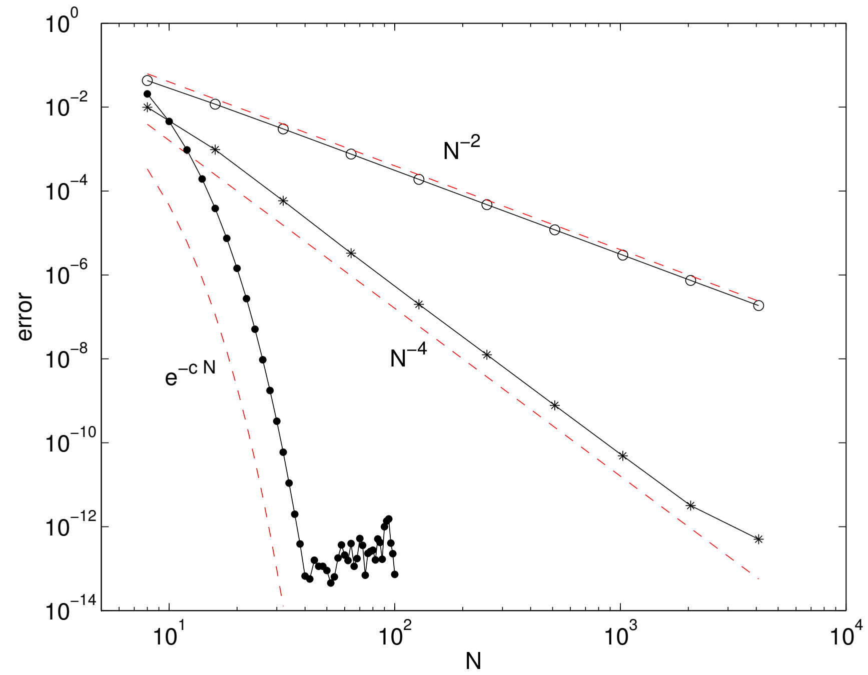
Accuracy Check or Error Convergence Test

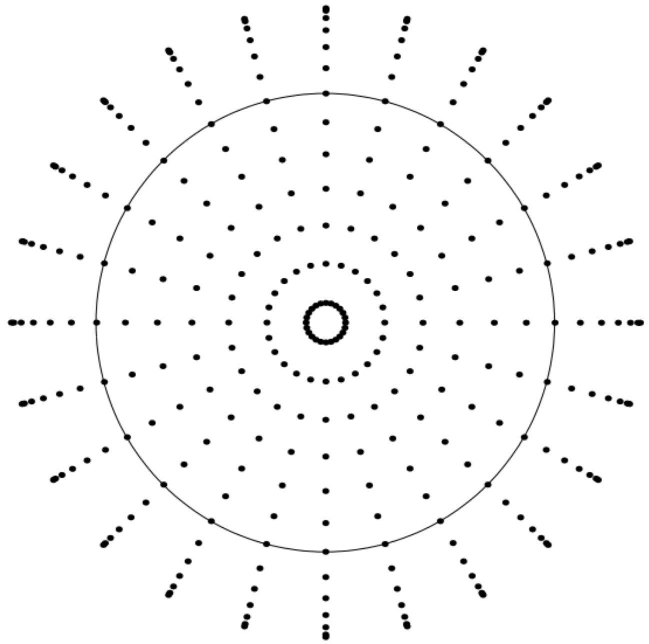
Chebyshev Points (Radial Direction) + Equally-Spaced Points (Angular Direction)

Used manufactured smooth solutions to

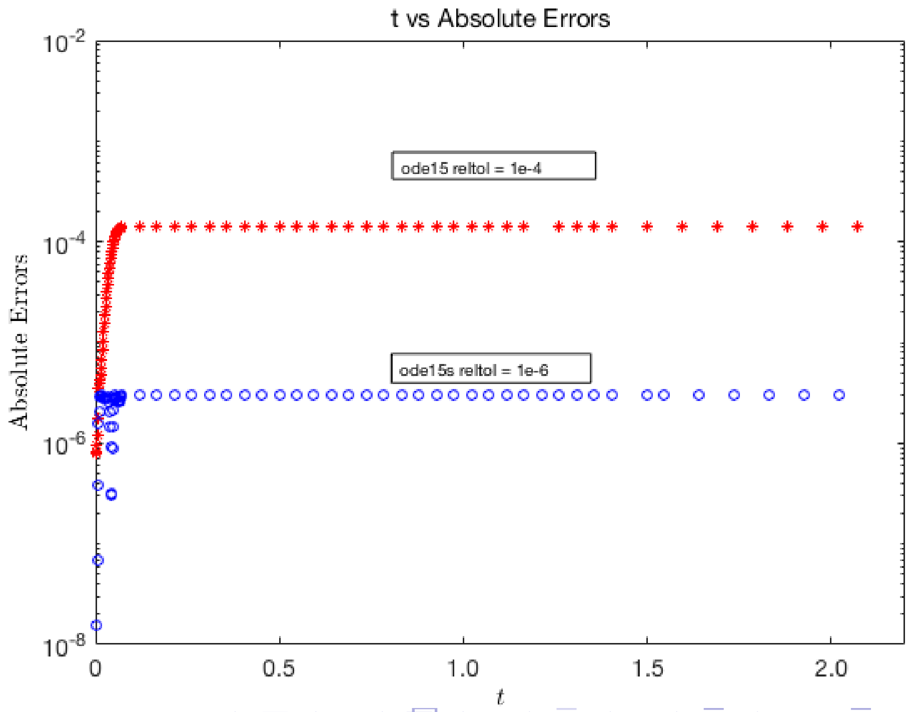
1. Check accuracy of all derivatives.
2. Check accuracy of Poisson solver.
3. Check accuracy of Nonlinear solver.

Differentiation matrices can be obtained from popular software package such as: Chebfun (Trefethen et al), Pseudopack (Don et al), DMSUITE (Weideman & Reddy)





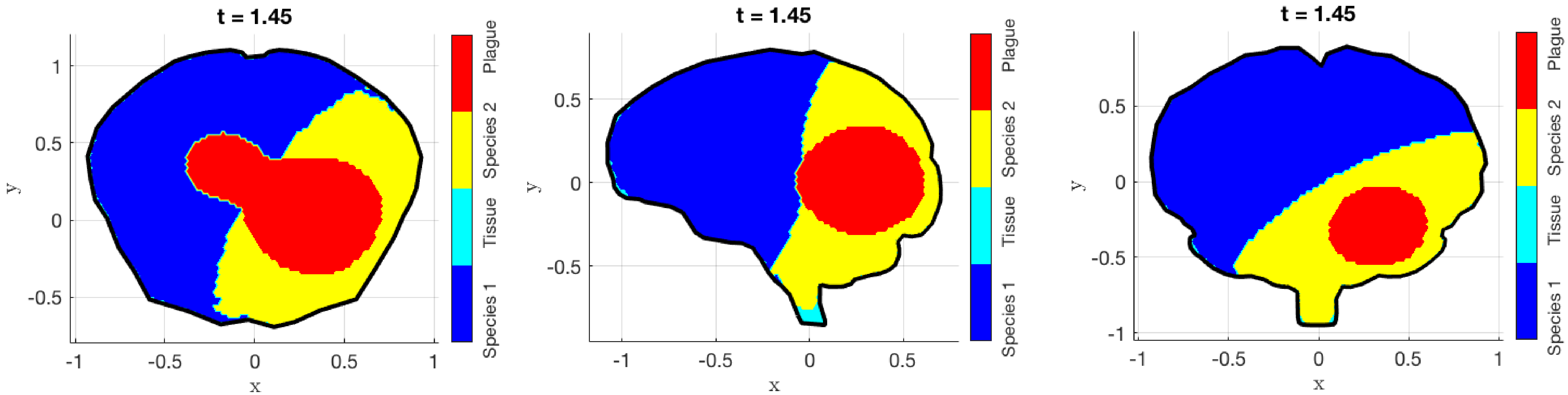
Fornberg's Polar Discretization



Numerical Simulation: Color Me Brain

Convention of Coloring: u_1: Amyloid species 1. u_2: Amyloid species 2.

- $u_1(x,y,t) > u_1^\varepsilon$: Blue. $u_2(x,y,t) > u_2^\varepsilon$: Yellow.
- $u_1(x,y,t) > u_2(x,y,t) \geq u_1^\varepsilon$: Bring blue to front.
- $u_2(x,y,t) > u_1(x,y,t) \geq u_2^\varepsilon$: Bring yellow to front.
- $u_1(x,y,t) + u_2(x,y,t) \geq U$: Plague: Red



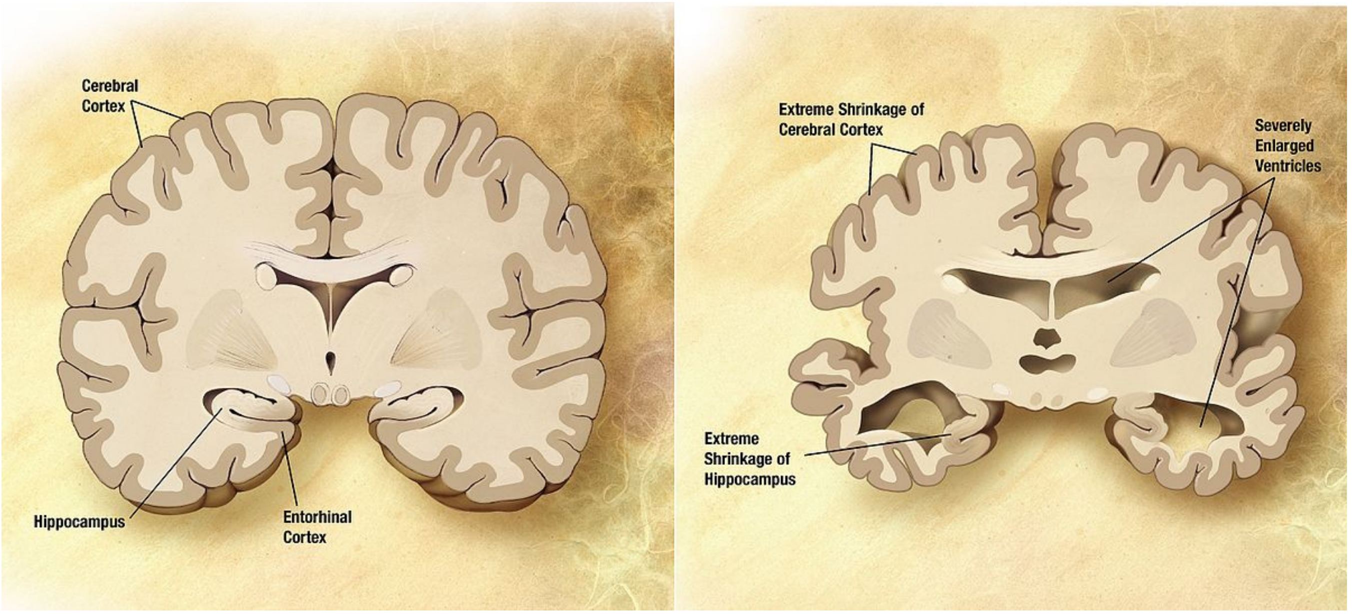

```
Tinit = 0; Tfin= 1.5;
opt   = odeset('RelTol',1e-8,'AbsTol',1e-10,'Jacobian', ...
                            @(t,u)Jodefun(t,u));
[T,U] = ode15s(@(t,u)odefun(t,u),[Tinit Tfin],uinit,opt);
```

Play/Pause

Play/Pause

Play/Pause

Trying on Frontal Lobe Cross-section (sort of)



Play/Pause

On-going study or future questions

Hopefully, we may get by with a little help from our friends.

- Dealing with multiple species. (speed up with MATLAB parallel computing toolbox (CPU and GPU), (Ferreira))
- Away from MATLAB ode solver. Experimenting with different time stepping methods, (Gottlieb, Ferreira).
- Conformal transplantation is cool (if you like it) but might not be a good way. Singularities due to $f'(z)$ might lead to numerical disaster. Meshfree collocation is considered.
- Dealing with multiply-connected domain with highly complex maze type pattern can lead to its own research.
- Dealing with shrinking domain (time-dependent conformal map). Another alternative is a totally space-time solver (Sousa, Field, Khanna)).
- Reduced basis method (Chen, Jiang, Bresten).

We want to thank UMass Dartmouth Center for Scientific Computing and Visualization Research for providing Rapid Prototyping Server for computation.