Analysis of the Du Fort–Frankel methods

Adi Ditkowski *, Neta Corem *

Abstract

The Du Fort–Frankel scheme was presented in 1953, [1], as a numerical method for solving the heat equation with periodic boundary conditions. This scheme was shown to be unconditionally stable with a truncation error of $O(\Delta t^2 + \Delta x^2 + \frac{\Delta t^2}{\Delta x^2})$. Therefore, when $\Delta t \approx \Delta x$ the scheme is not consistent.

In the 1970's and 1980's this method was generalized for solving general initial–boundary value problems, and as a time propagating scheme for high order spatial approximations. The generalized scheme was also shown to be unconditionally stable where a dominating term in truncation error is $O(\frac{\Delta t^2}{\Delta x^2})$.

In this talk we prove that the Du Fort–Frankel methods are consistent when $\Delta t,\ \Delta x \ll 1$. However, when $\Delta t = O(\Delta x)$ these schemes are not stable.

References

[1] E. C. Du Fort and S. P. Frankel, *Conditions in the Numerical Treatment of Parabolic Differential Equations*, Mathematical Tables and Other Aids to Computation, Vol. 7, No. 43, 1953), pp.135-152 (1953).

*School of Mathematical Sciences, Tel Aviv University, Tel Aviv 69978, Israel
*School of Mathematical Sciences, Tel Aviv University, Tel Aviv 69978, Israel