

High Accuracy Solution of the Helmholtz Equation in General Shaped Domains and Interfaces

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We consider the numerical solution of the Helmholtz equation for the domains where the boundaries and interfaces may not necessarily conform to the mesh. We consider a geometrically large region of space separated by several arbitrarily shaped interfaces and truncated with an arbitrarily shaped external boundary. The material properties are assumed smooth between the interfaces, whereas at the interfaces they may undergo discontinuities. Due to pollution, it is strongly desirable that higher order methods be used to reduce the errors, especially for high frequencies. However, the existence of interfaces usually degrades the accuracy of the scheme.

The new technique will use only simple structured grids, e.g., Cartesian or polar. In the regions of smoothness, we employ high order compact finite difference schemes. The boundaries and interfaces that are not aligned with the grid are treated by Calderon operators and the method of difference potentials. This involves no loss of accuracy for arbitrarily shaped boundaries. The boundary representations inherit the accuracy of the core scheme, and high order accuracy is achieved for non-conforming boundaries. No adverse effects due to staircasing occur on regular structured grids with curved boundaries. Variable coefficients present no difficulties. The procedure is automatic and the discrete equations are fully characterized in algorithmic terms. It is possible to prove that the equivalent boundary problem that involves the Calderon projection is always well-posed.

We will present results for a fourth accurate solution of the Helmholtz equation using a Cartesian grid for non-aligned bodies. Both interior and exterior problems will be considered.