High Order Weighted Essentially Non-Oscillatory WENO-Z schemes for Hyperbolic Conservation Laws

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The authors have designed a new fifth order WENO finite-difference scheme by adding a higher order smoothness indicator which is obtained as a simple and inexpensive linear combination of the already existing low order smoothness indicators. Moreover, this new scheme, dubbed as WENO-Z, has a CPU cost which is equivalent to the one of the classical WENO-JS and significantly lower than that of the mapped WENO-M, since it involves no mapping of the nonlinear weights. In this article, we take a closer look at Taylor expansions of the Lagrangian polynomials of the WENO sub-stencils and the related inherited symmetries of the classical lower order smoothness indicators to obtain a general formula for the higher order smoothness indicators that allows the extension of the WENO-Z scheme to all (odd) orders of accuracy. We further investigate the improved accuracy of the WENO-Z schemes at critical points of smooth solutions as well as their distinct numerical features as a result of the new sets of nonlinear weights and we show that regarding the numerical dissipation WENO-Z occupies an intermediary position between WENO-JS and WENO-M. Some standard numerical experiments such as the one dimensional Riemann initial values problems for the Euler equations and the Mach 3 shock density-wave interaction and the two dimensional double-Mach shock reflection problems are presented. Some works on the detonation waves and shock particle-laden flows are also presented.

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