SL1. Rodrigo Platte

Title: Convergence properties of analytic and C^∞ compactly-supported radial basis function interpolation

Abstract: We present a framework for estimating convergence rates of radial basis function approximations of analytic functions and explore the susceptibility of analytic kernels to the Runge phenomenon. Stable interpolation in this case is restricted to special nodal sets or carefully chosen shape parameters. As an alternative to analytic kernels, we explore a family of C^∞ compactly-supported radial basis functions. By compromising geometric convergence, better conditioned schemes with sparser matrices can be constructed.

SL2. Grady Wright

Title: An algorithm for stable computations with flat radial basis functions

Abstract: Smooth radial basis functions (RBFs) are receiving considerable attention for applications such as interpolation and numerically solving partial differential equations (PDEs) since they combine high-order/spectral accuracy with meshless flexibility. These RBFs feature a shape parameter and one commonly finds in applications that using shape parameters which produce "flat" RBFs result in smaller discretization errors. However, the direct numerical approach (referred to as RBF-Direct in some present literature) of computing with flat RBFs is severely ill-conditioned. Until recently, there was a widespread misconception that this flat regime was therefore computationally impracticable. The first algorithm to prove this misconception wrong and allow for stable computations for all shape parameters was the Contour-Padé (CP) method (a second method called RBF-QR has since been developed). In this talk we discuss how the original RBF-CP method can be greatly improved in terms of accuracy, robustness, and automated selection of parameters. The improvements come from replacing the Padé-part of the method with a more directly determined rational approximation (RA). We present the method (which we call RBF-RA) together with some numerical examples demonstrating its accuracy, robustness, and applicability to different RBF based methods.

SL3. Greg Fasshauer

Title: Computing with Reproducing Kernels

Abstract: Positive definite kernels and their associated reproducing kernel Hilbert spaces provide a very flexible and powerful tool for the solution of many typical problems of numerical analysis and scientific computing such as function approximation, numerical integration or the numerical solution of PDEs. I will provide an introduction to positive definite reproducing kernels, mention some applications, and end with a discussion of some directions that may lead to potential research topics for interested students.

SL4. Sigal Gottlieb

Title: Gegenbauer postprocessing for radial basis function simulations of hyperbolic PDEs

Abstract: RBF methods are global methods which do not require the use of specialized points and that yield high order accuracy if the function is smooth enough. Like other global approximations, the accuracy of RBF approximations of discontinuous problems deteriorates due to the Gibbs phenomenon, even as more points are added. In this talk we show that it is possible to remove the Gibbs phenomenon from RBF approximations of discontinuous functions as well as from RBF solutions of some hyperbolic partial differential equations. Although the theory for the resolution of the Gibbs phenomenon by reprojection in Gegenbauer polynomials relies on the orthogonality of the basis functions, and the RBF basis is not orthogonal, we observe that the Gegenbauer polynomials recover high order convergence from the RBF approximations of discontinuous problems in a variety of numerical examples including the linear and nonlinear hyperbolic partial differential equations. Our numerical examples using multi-quadric RBFs suggest that the Gegenbauer polynomials are *Gibbs complementary* to the RBF multi-quadric basis.

SL5. Jae-Hun Jung

Title: Radial basis function methods for discontinuous problems

Abstract: Radial basis function (RBF) approximations for discontinuous functions suffer from the Gibbs phenomenon that yields spurious oscillations near the local discontinuity and may cause instability in time. In this talk, we will explain what we have done so far to deal with discontinuous problems for the RBF approximation. First, we explain what we observed that a proper postprocessing method such as the Gegenbauer reconstruction method recovers high order convergence from the RBF approximations of discontinuous problems in a variety of numerical examples. Our numerical examples using multiquadric RBFs suggests that the Gegenbauer polynomials are Gibbs complementary to the RBF multiquadric basis. Second, we explain the iterative adaptive RBF method for the local edge detection both in one and two space dimensions. The iterative method detects edges successively by changing the shape parameter adaptively. Numerical results show that the method finds edges in an accurate manner when the RBF approximation defined on the grid (centers). Then we will explain the interface problem and the least-squares RBF method. We will compare our results with the spectral method. We observe that the least-squares spectral method is superior to the least-squares RBF method in terms of accuracy. We will discuss some possible suggestions to improve the results.

SL6. Toby Driscoll

Title: Chebfun: A software system for interacting with functions

Abstract: The Chebfun software package provides a natural extension of Matlab's handling of vectors and matrices to one-dimensional functions and operators on them. Using fast, stable, and accurate algorithms based on Chebyshev polynomial interpolation, Chebfun provides automatic facilities for computations on smooth and piecewise smooth functions and differential equations. The package includes a graphical interface for exploring the solutions of boundary-value problems, eigenvalue problems, and partial differential equations.