Radial Basis Functions for Computational Geoscience

Natasha Flyer and Bengt Fornberg

Modeling Motivations for RBF Computational Research

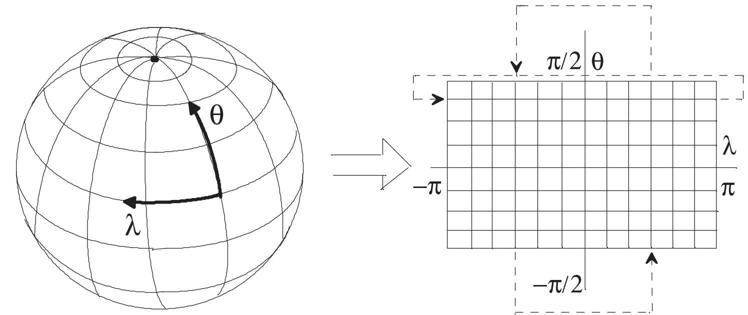
Examples:

- Easy model coupling
- Necessary scalability
- Free-boundary problems
- Geometric flexibility
- Algorithmic simplicity
- Long term time stability
- Resolvable temporal and spatial scales that validate the physics

Bottom Line

High-resolution and numerical accuracy at low computational costs to resolve the multi-scale features of physical flows

Highlights of Some High-Order Methods in Large-Scale Models



Double Fourier series:

Strength: Exponential accuracy
Computationally fast because of FFTs

Weakness: No practical option for local mesh refinement

Spherical harmonics:

Strength: Exponential accuracy

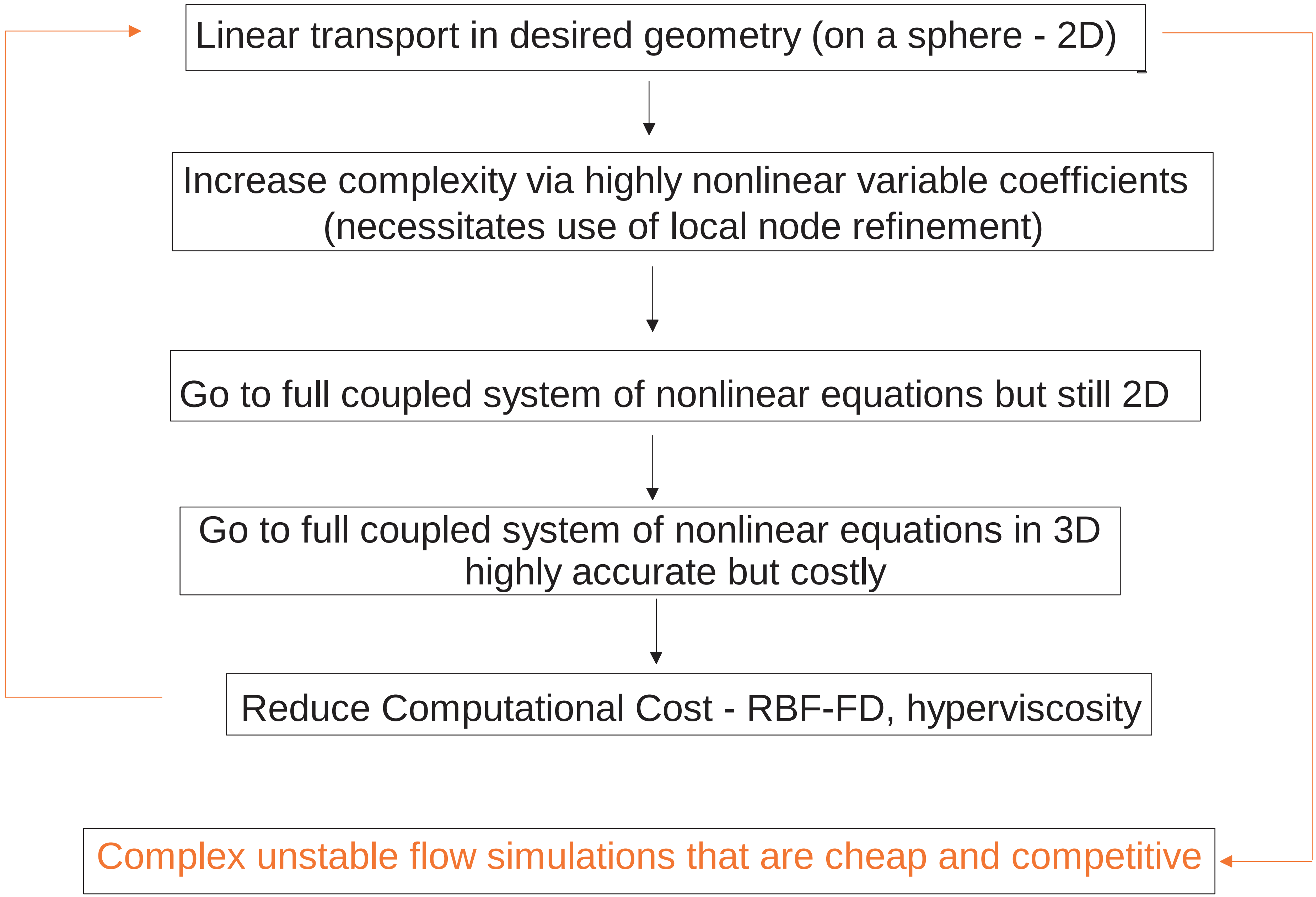
Weakness: No practical option for local mesh refinement
Relatively high computational cost
Poor scalability on parallel computer architectures

Spectral elements:

Strength: Accuracy approaching exponential
Local refinement is feasible but complex

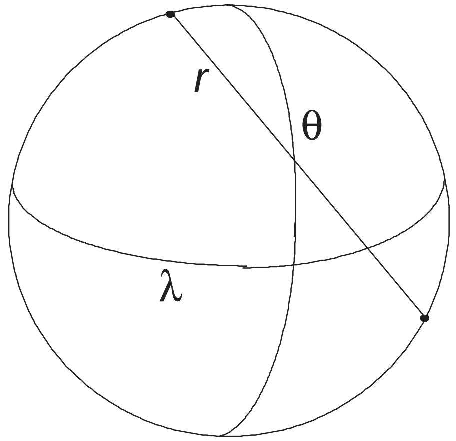
Weakness: Loss of efficiency due to unphysical element boundaries
(Runge phenomenon - oscillations near boundaries → restrictive time-step)
High algorithmic complexity
High pre-processing cost

Flowchart for Developing Large-Scale RBF Convective Models



RBFs: The Gradient Operator

(Flyer and Wright, *JCP*, 2007)



$$\nabla = \frac{1}{\cos\theta}\frac{\partial}{\partial\lambda}\hat{\lambda} + \frac{\partial}{\partial\theta}\hat{\theta}$$

Singular at the poles $\theta = \pm\frac{\pi}{2}$ unless $\frac{\partial}{\partial\lambda}$ also vanishes

$$r = ||\underline{x} - \underline{x}_j|| = \sqrt{2}\sqrt{1 - \cos\theta\cos\theta_j\cos(\lambda-\lambda_j) - \sin\theta\sin\theta_j}$$

$$\frac{\partial}{\partial\lambda}\phi = \frac{\sqrt{2}\cos\theta\cos\theta_j\sin(\lambda-\lambda_j)}{r}\frac{\partial\phi}{\partial r}$$

$$\frac{\partial}{\partial\theta}\phi = \sqrt{2}\,\frac{\sin\theta\cos\theta_j\cos(\lambda-\lambda_j) - \cos\theta\sin\theta_j}{r}\frac{\partial\phi}{\partial r}$$

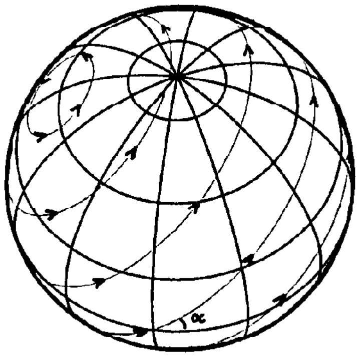
$$\boxed{\nabla\phi = \left[\cos\theta_j\sin(\lambda-\lambda_j)\hat{\lambda} + \left[\sin\theta\cos\theta_j\cos(\lambda-\lambda_j) - \cos\theta\sin\theta_j\right]\hat{\theta}\right]\frac{\sqrt{2}}{r}\frac{\partial\phi}{\partial r}}$$

Notice that <u>nowhere</u> is the gradient operator singular !

No pole singularities even though spherical coordinates are used!

Solid Body Rotation of C [1] Cosine Bell

(Flyer and Wright, JCP, 2007)



Method of lines formulation

$$\frac{\partial h}{\partial t} = -\left(\underline{U} \bullet \nabla\right) h \quad \Leftrightarrow \quad \frac{\partial h}{\partial t} = -D_N h$$

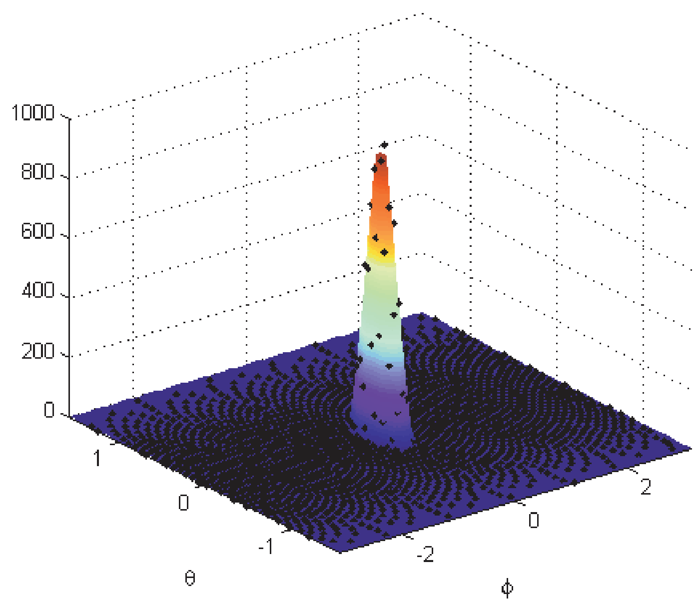
D_N is discrete RBF differentiation matrix:

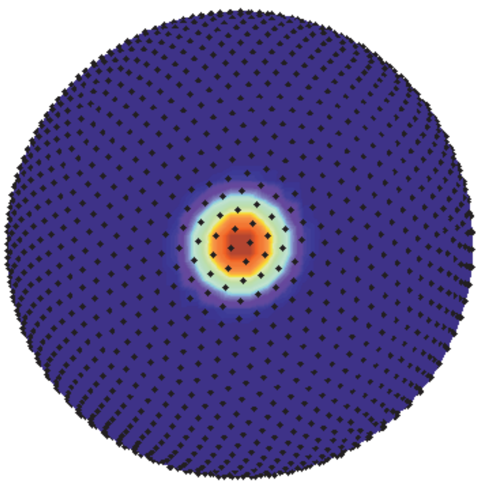
- Free of Pole Singularities (although posed in spherical coordinates)
- Error Invariant to angle of rotation

Gaussian RBFs

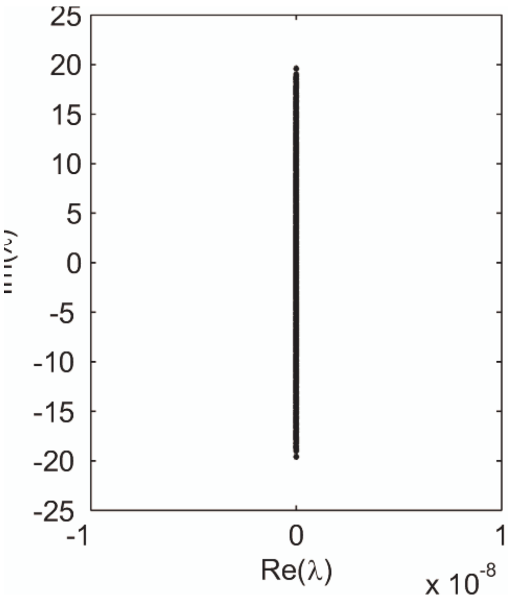
$N = 1849$ nodes, $\varepsilon = 6$

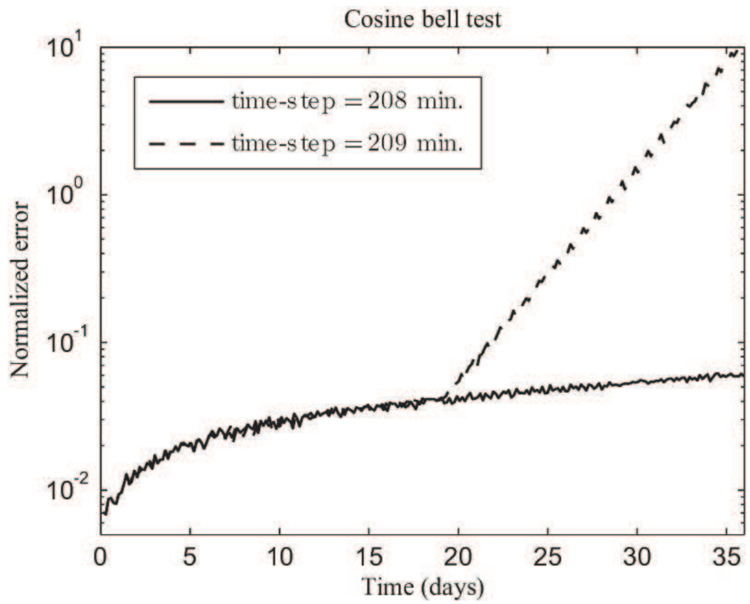
$\Delta t = 45$ minutes; 4^{th} order Runge-Kutta

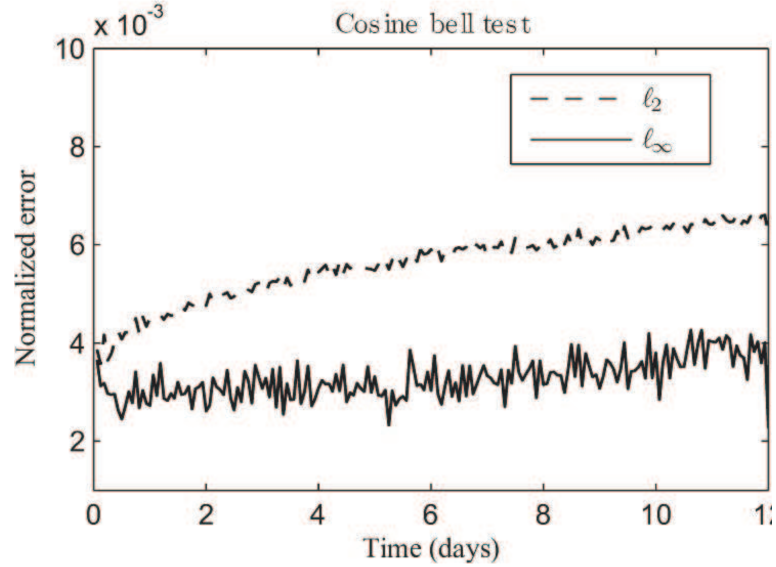




Stability Analysis







Eigenvalues of D_N exactly on imaginary axis

Eigenvalues have to fit inside RK4's stability domain.

For best computational efficiency choose time steps so that time and space errors match

D_N is a product of a positive definite matrix and an antisymmetric matrix (Platte and Driscoll, 2006)

When largest stable time steps are used, time errors dominate

Here, N = 4096 the time step is **30 minutes** over space errors.

Comparison between main spectral methods on the sphere

For a ℓ_2 Error of 0.005:

Method	N = # of nodes	Time Step	Local Mesh Refinement	Cost per Time Step
RBF	4096	30 minutes	Yes	$O(N^2)$
SH	32,768	90 seconds	No	$O(N^{3/2})$
DF	32,768	90 seconds	No	$O(N \log N)$
DG/SE	7776	6 minutes	Yes	$O(k\,M)$

Comments: - RBF code 37 lines in MATLAB using no built-in subroutines

Spectral Elements: k = number of elements
M = number of nodes/element

Spherical Harmonics: a_k = 7396 harmonics

T170 (N=131,072 , $\Delta t = 7.5$ min. semi-implicit)

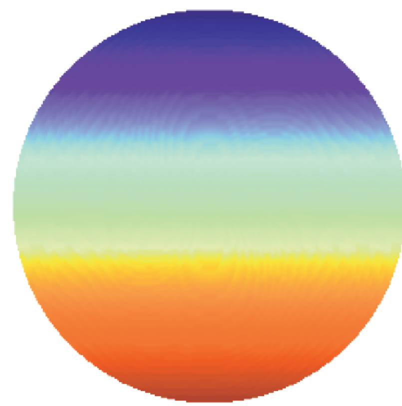
Code: Appendix B Flyer and Wright, JCP, 2007

```
ep      =  6;                  %   Value of epsilon
R       = 1/3;                  %  Width of bell on unit sphere
alpha = pi/2;                   %  Angle of rotation measured from the equator
%%% Load Nodes: http://web.maths.unsw.edu.au/~rsw/Sphere/Energy/index.html and compute r^2  %%%

load('me1849.dat');   x = me1849(:,1);   y = me1849(:,2);   z = me1849(:,3);          % Cartesian
theta = atan2(z,sqrt(x.^2+y.^2));       tt   =  theta(:,ones(length(theta), 1));              % latitude    - spherical
phi   =  atan2(y,x);                         pp =   pn(:,ones(length(phi), 1));                    % longitude - spherical
r2     = 2 * (1 - cos(tt').*cos(tt).*cos(pp' -pp) - sin(tt').*sin(tt));

 %%% Compute differentiation matrix D %%%%

B = (cos(alpha).*cos(tt).*cos(tt').*sin(pp-pp') + sin(alpha).*(cos(tt).*cos(pp).*sin(tt') -cos(tt').*cos(pp').*sin(tp)))
B = 12 * pi * B.*(-ep^2*exp(-ep^2.*r2));
A = exp(-ep^2.*r2);
D = B/A;
%%% Initial Condition Cosine Bell %%%

h = 1000/2*(1+cos(pi*(acos(cos(theta).*cos(phi)) )/R));
h(acos(cos(theta).*cos(phi)) >= R) = 0;

%%%  Classic 4th Order RK %%%

dt           =  12/288*5/6;         % Time step for 12 day revolution
for  nt    =   2:(1*288*6/5)
    d1 =   dt*D*h;
    d2 =   dt*D*(h + 0.5*d1);
    d3 =   dt*D*(h + 0.5*d2);
    d4 =   dt*D*(h + d3);
      h =    h + 1/6*(d1 + 2*d2 + 2*d3 + d4);
end
```

Moving Vortex Roll-Up on A Sphere: Local Node Refinement

(Flyer & Wright, *JCP*,2007, Flyer & Lehto, *JCP*, 2010)

Initial condition



Solution after 12 days

Linear convection with a vortex-like flow field

(wind field is time-dependent)

Numerical implementation

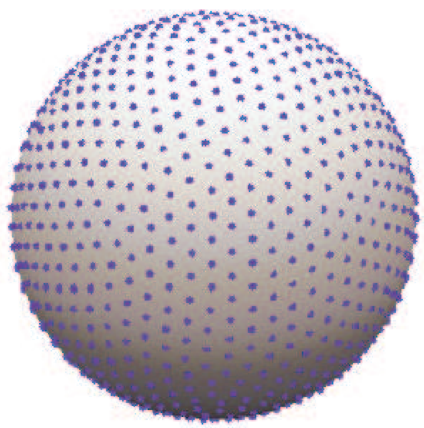
IMQ RBFs: $\phi(r) = \dfrac{1}{\sqrt{1+\varepsilon^2 r^2}}$

N = 3136 nodes

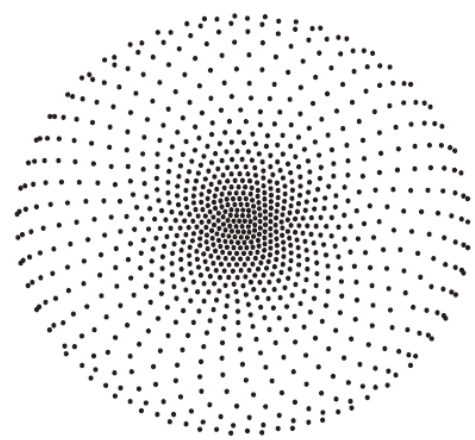
(1849 shown in figures to the right)

Method of lines (MOL) time stepping with standard Runge-Kutta, 4th order

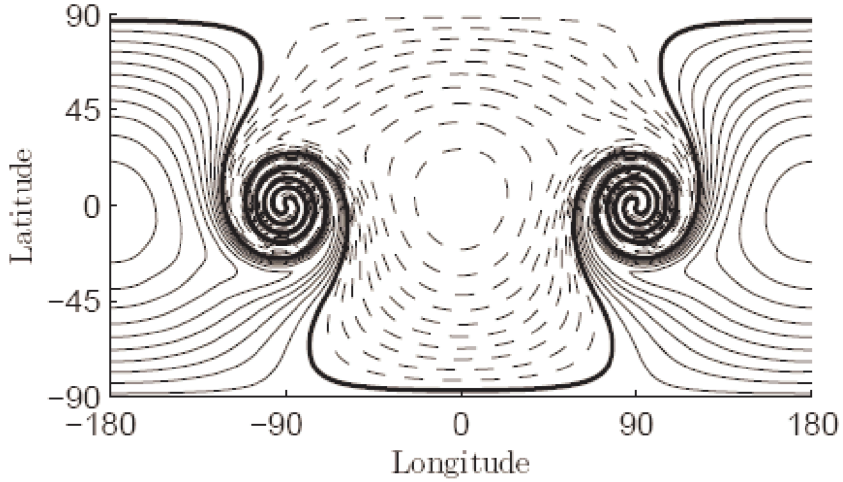
Minimal energy (ME) nodes



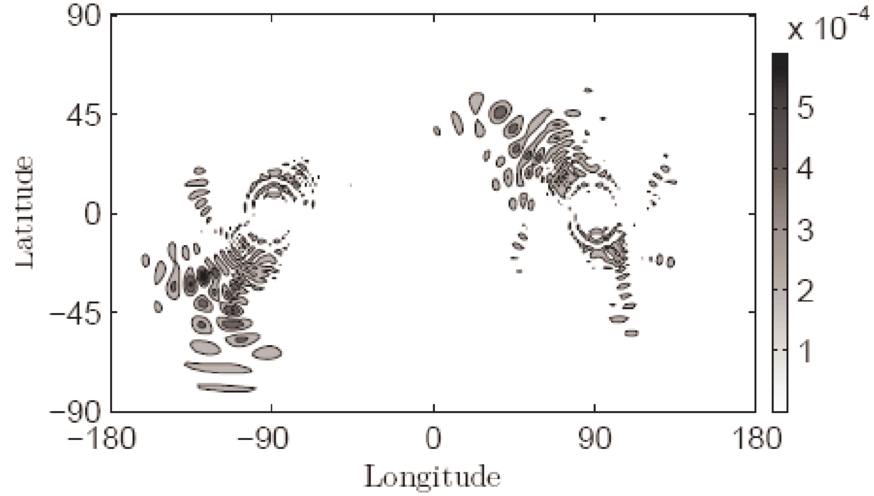
Refined nodes



RBF solution at 12 days, N=3136



Error at 12 days (10^{-4})



Method		Resolution N (total)	Resolution Typical angular	Time step Minutes	Error l_2 error
	Without local refinement				
RBF	Radial basis functions	3,136	6.4 ^{o}	(2hr) 60	4×10^{-3}
FV	Finite Volume (cubed sphere)	38,400	1.125 ^{o}	30	2×10^{-3}
DG	Discontinuous Galerkin	9,600	2.6 ^{o}	6	7×10^{-3}
	With local refinement				
RBF	Radial basis functions	3,136	-	(50) 20	8×10^{-5}
FV	Finite Volume (3 levels.lat-long)	-	5 ^{o} - 0.625 ^{o}	1-3	2×10^{-3}

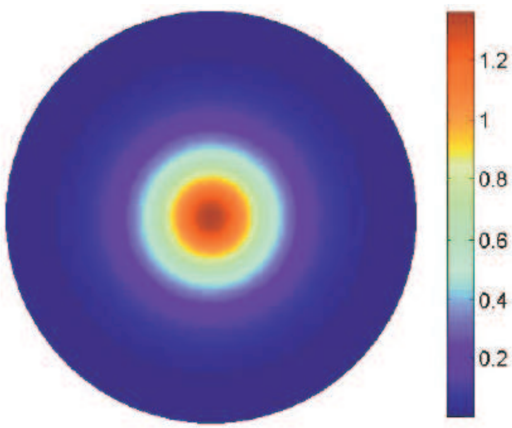
For the same accuracy, RBFs use less node points with larger time steps

Variable Shape Parameter of RBF, Epsilon ε

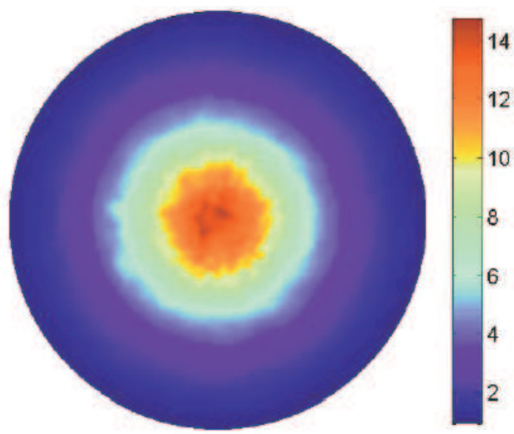
(Fornberg and Zuev, 2007)

When clustering nodes, the shape parameter must be vary to avoid Runge Phenomena

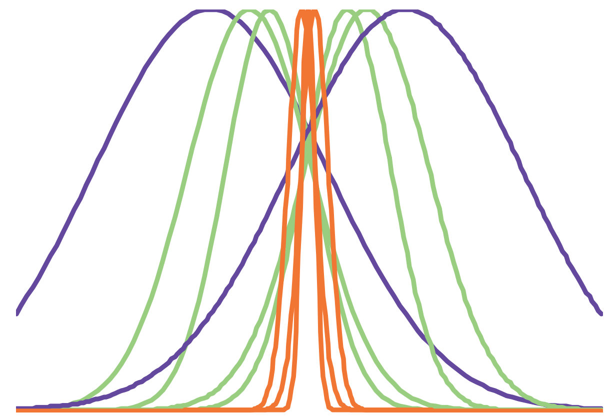
Heuristic: Inverse of Euclidean distance to nearest neighbor node



ω



$d_j = 1/\ell_2$ distance to nearest neighbor



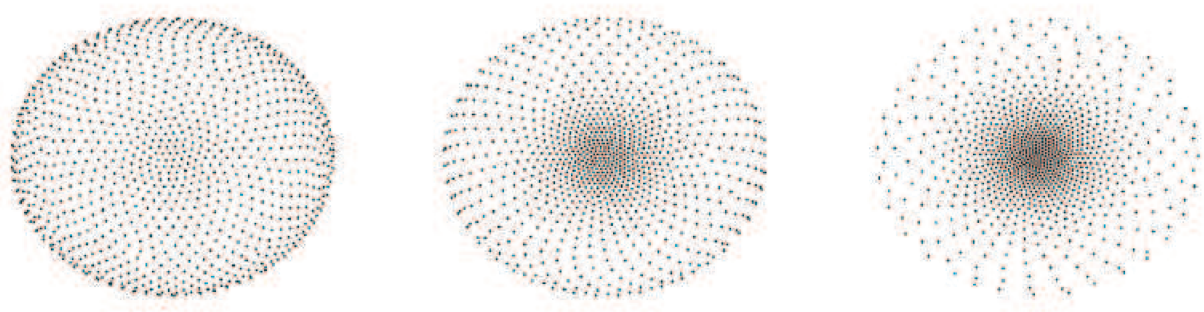
Cross-section view of RBFs

$$\boxed{\varepsilon_{j\,(\varphi_j,\theta_j)} = \varepsilon \frac{\max_{all\,j} d_j}{d_j}}$$

optimal value for $\varepsilon \sim O(1)$

Local Refinement and Matrix Conditioning

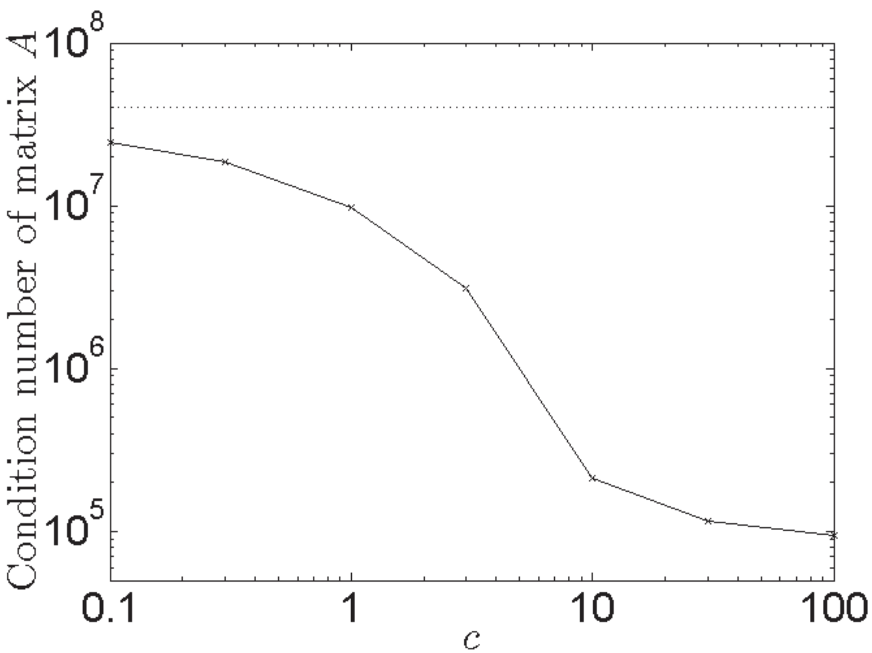
In refinement scheme, 'c' is a parameter that controls the amount of clustering

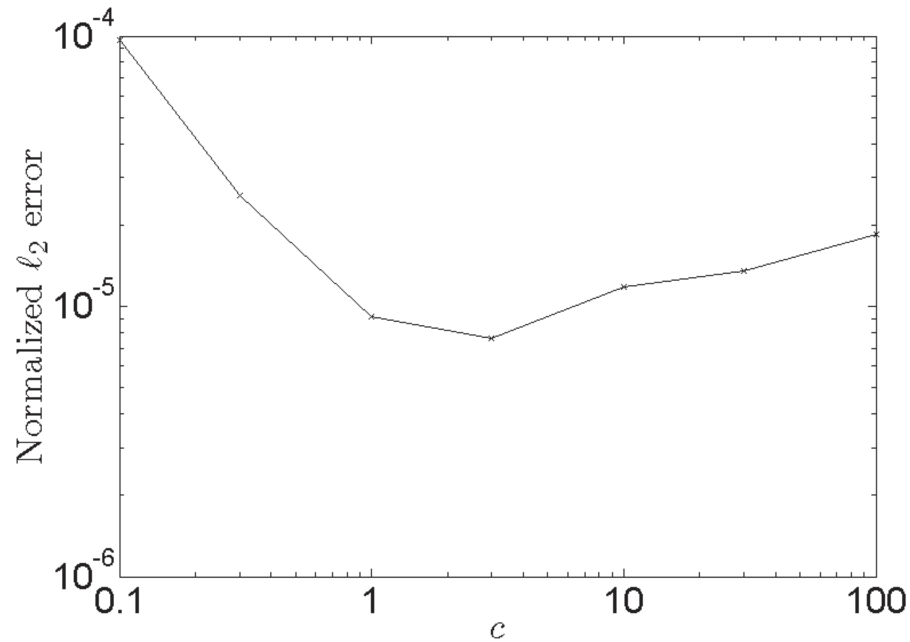


c = 0.1 c = 1 c = 10

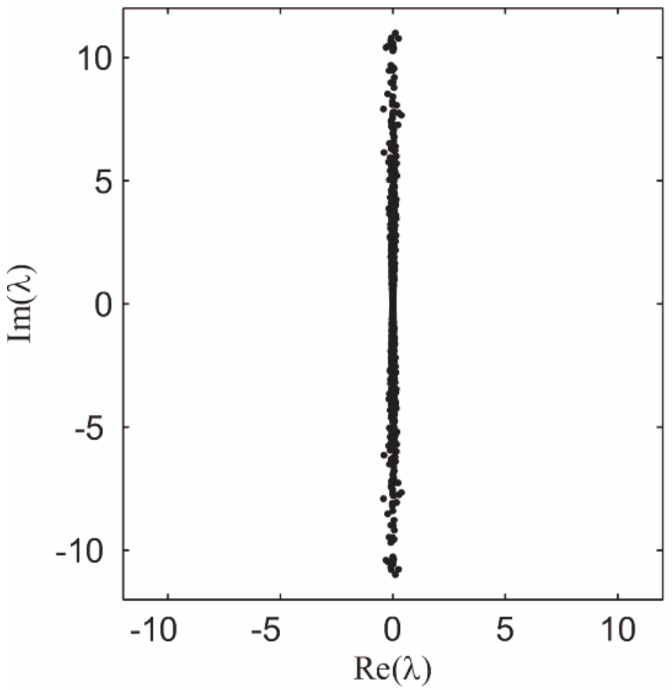
Clustering nodes → the shape parameter must vary to avoid Runge Phenomena →

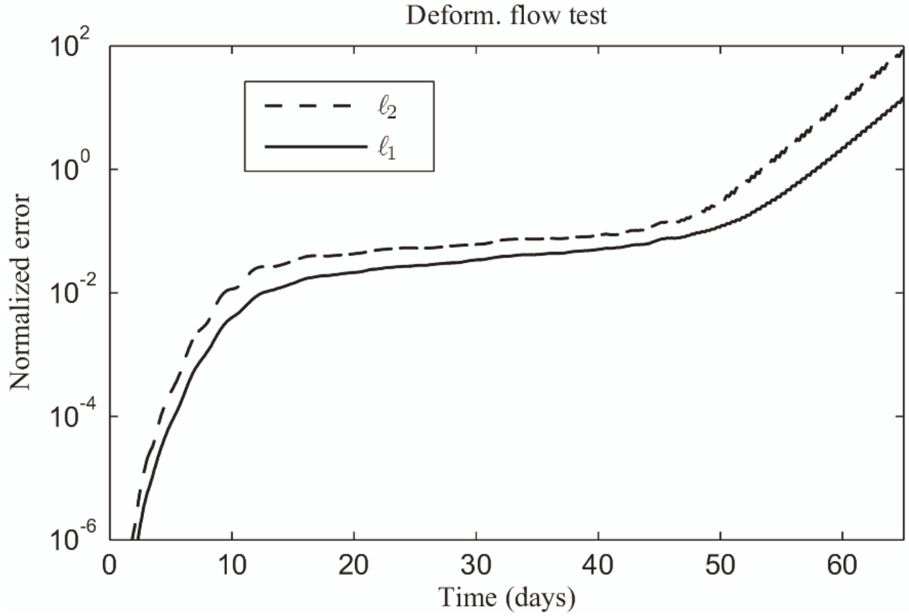
Inverse of Euclidean distance to nearest neighbor



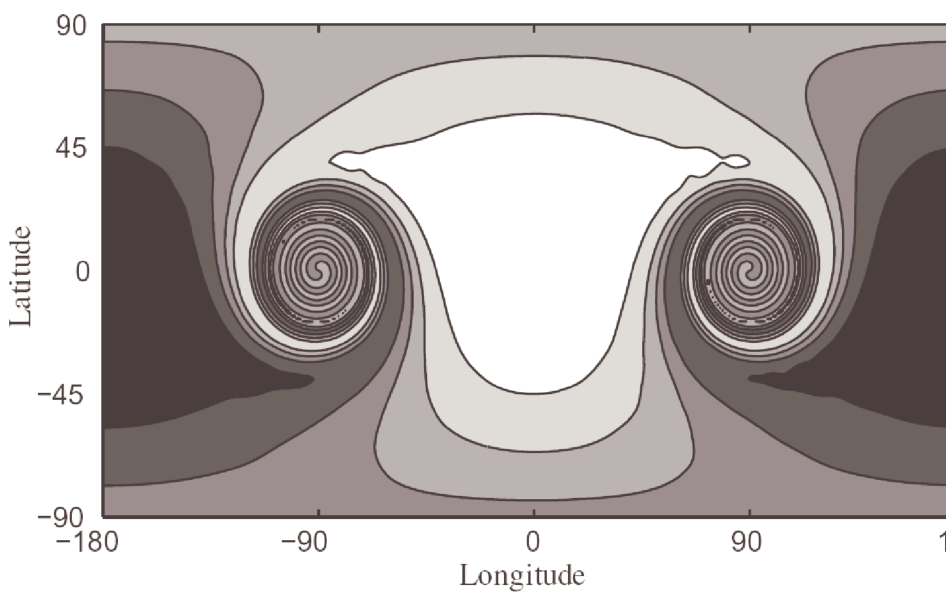


Eigenvalue Study

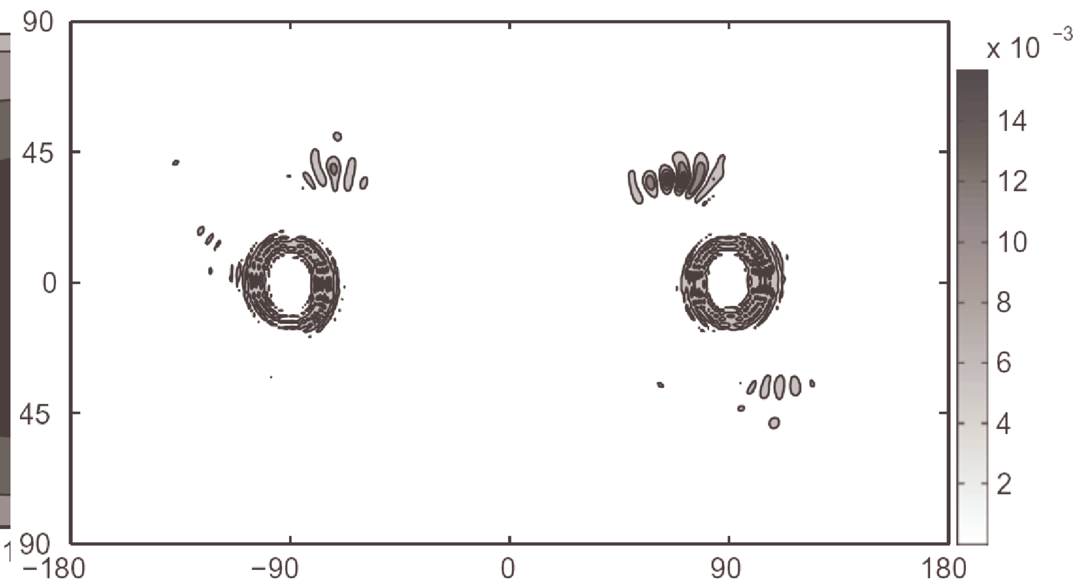




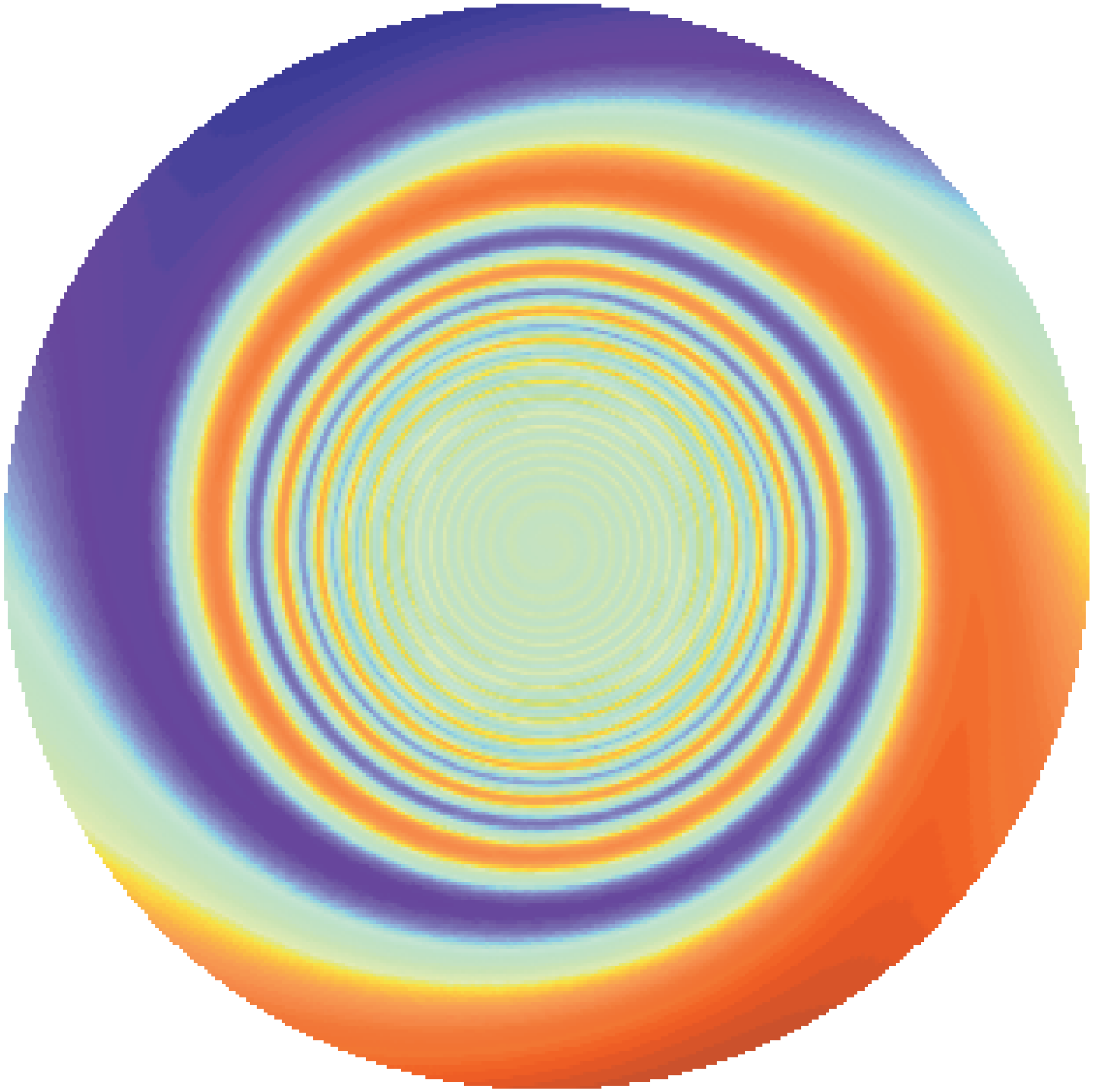
Solution at 24 Days, N = 3136



Error at 24 Days , N = 3136 (10^{-3})



Exact Solution at 45 Days



- No adverse effects of positive real parts until solution features have become too fine to be resolvable (theoretical limit 2 nodes / wave length)

Go Nonlinear - Shallow Water Equations on a Sphere (Cartesian)

(Flyer and Wright, *Proc. Roy. Soc. A*, 2009)

Acceleration Advection Coriolis Pressure

$$\frac{\partial u}{\partial t} = -\mathbf{p}_x \cdot \underbrace{\begin{bmatrix} (\mathbf{u}\cdot\mathbf{P}\nabla)u + f(\mathbf{x}\times\mathbf{u})\cdot\hat{\imath} + g(\mathbf{p}_x\cdot\nabla)h \\ (\mathbf{u}\cdot\mathbf{P}\nabla)v + f(\mathbf{x}\times\mathbf{u})\cdot\hat{\jmath} + g(\mathbf{p}_y\cdot\nabla)h \\ (\mathbf{u}\cdot\mathbf{P}\nabla)w + f(\mathbf{x}\times\mathbf{u})\cdot\hat{\mathbf{k}} + g(\mathbf{p}_z\cdot\nabla)h \end{bmatrix}}_{\mathrm{RHS}},$$

$$\frac{\partial v}{\partial t} = -\mathbf{p}_y \cdot \mathrm{RHS}, \qquad \frac{\partial w}{\partial t} = -\mathbf{p}_z \cdot \mathrm{RHS}, \qquad \frac{\partial h}{\partial t} = -(\mathbf{P}\nabla)\cdot(h\mathbf{u})$$

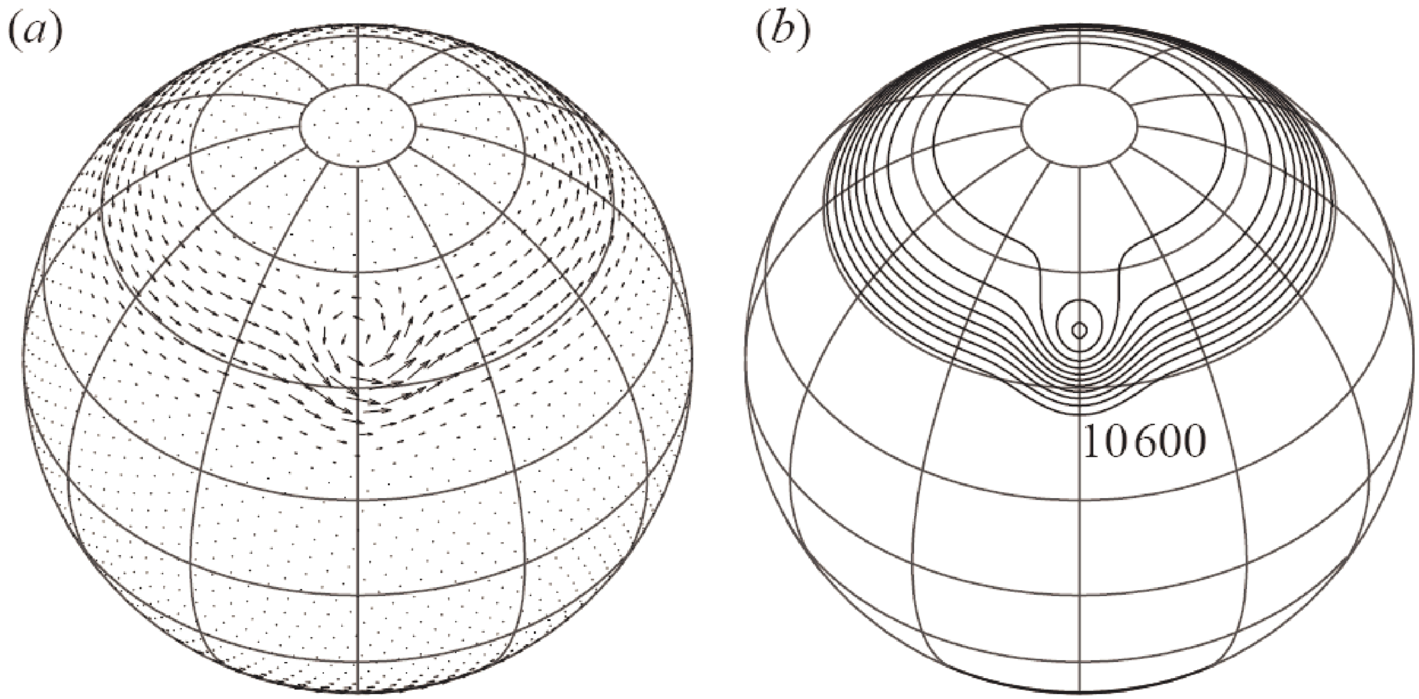
Sphere Projection Matrix $\mathbf{P}$

$$\mathbf{P} = \begin{bmatrix} \underline{p_x} \\ \underline{p_y} \\ \underline{p_z} \end{bmatrix} = \begin{bmatrix} (1-x^2) & -xy & -xz \\ -xy & (1-y^2) & -yz \\ -xz & -yz & (1-z^2) \end{bmatrix}$$

Forced Translating Low Pressure System: RBF Shallow Water Model

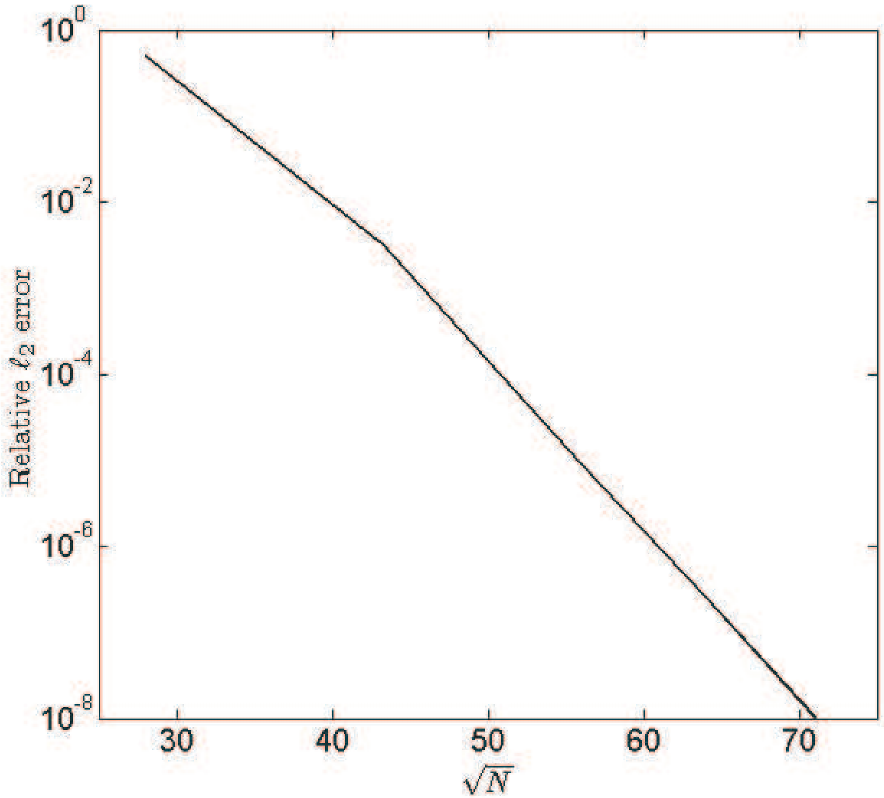
Forcing terms added to the shallow water equations to generate a flow that mimics a short wave trough embedded in a westerly jet.

Initial Velocity Field　　　　Initial Geopotential Height Field

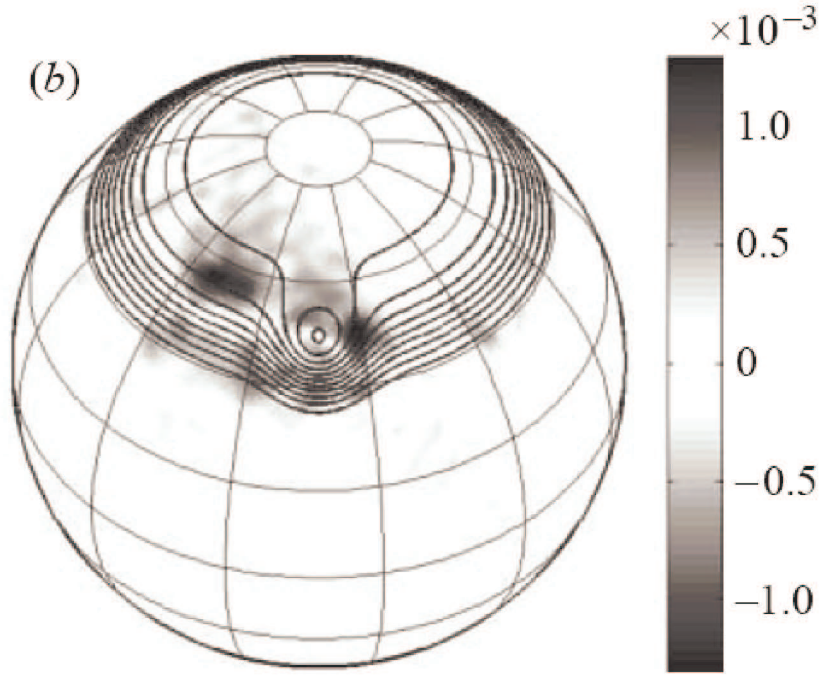


Errors after Wave Trough Has Traveled 5 Days

Convergence rate in the ℓ_2 norm



$N = 3136$ nodes, $\Delta t = 15$ minutes



Relative Difference in Mass and Energy after:

Mass	5 days	25 days	Energy	5 days	25 days
N=3136	2×10^{-9}	4×10^{-9}		-3×10^{-9}	2×10^{-9}
N=4096	1×10^{-11}	-2×10^{-10}		-1×10^{-10}	-5×10^{-10}

Comparison with Commonly Used Methods

Method	N	Time step	Relative ℓ_2 error
RBF	4,096	8 minutes	2.5×10^{-6}
	5,041	6 minutes	1.0×10^{-8}
Sph. Harmonic	8,192	3 minutes	2.0×10^{-3}
Double Fourier	32,768	90 seconds	4.0×10^{-4}
Spect. Elem.	24,576	45 seconds	4.0×10^{-5}

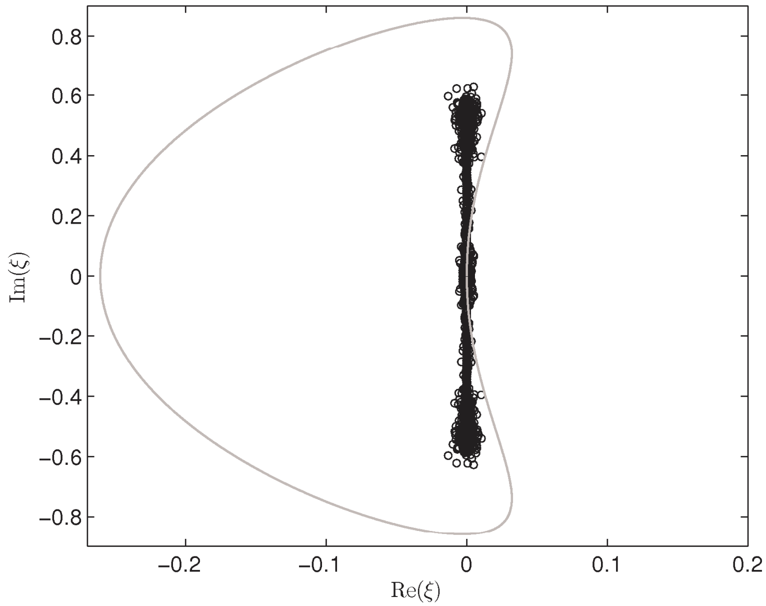
Time Step for RBF: Temporal Errors = Spatial Errors
Time Step for Others: Stability Limited

RBF Computational times, in Matlab on 2.66 GHz Single Core Processor

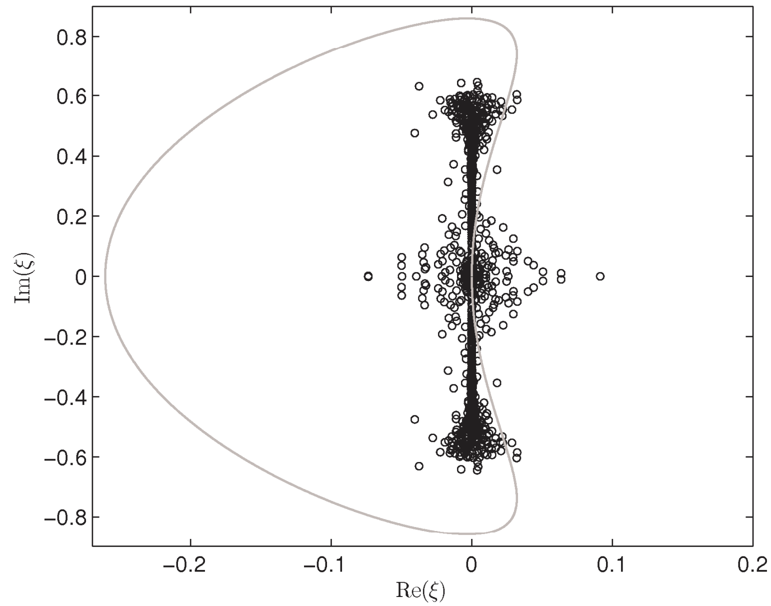
N	Runtime per time step (sec)	Total Runtime
4,096	0.41	6 minutes
5,041	0.60	12 minutes

For much higher numerical accuracy, RBFs uses less nodes & larger time steps

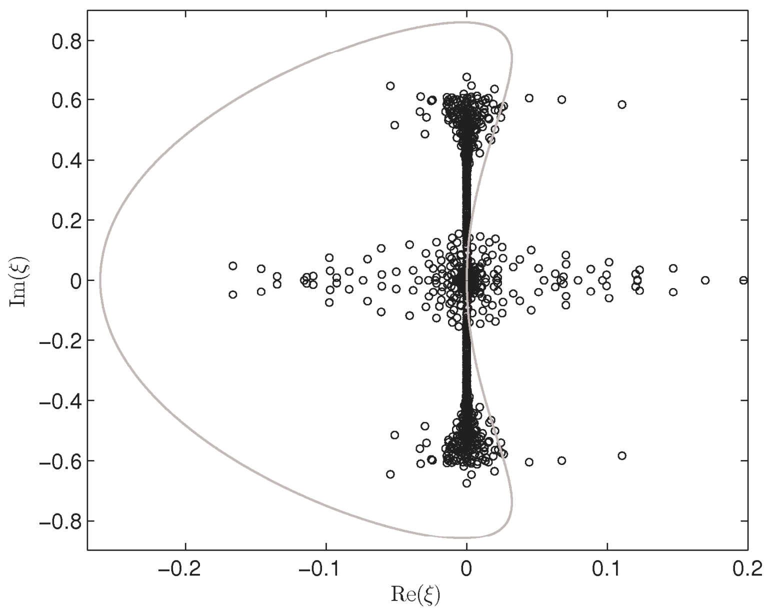
Eigenvalues, Time Stability, and ε - refinement



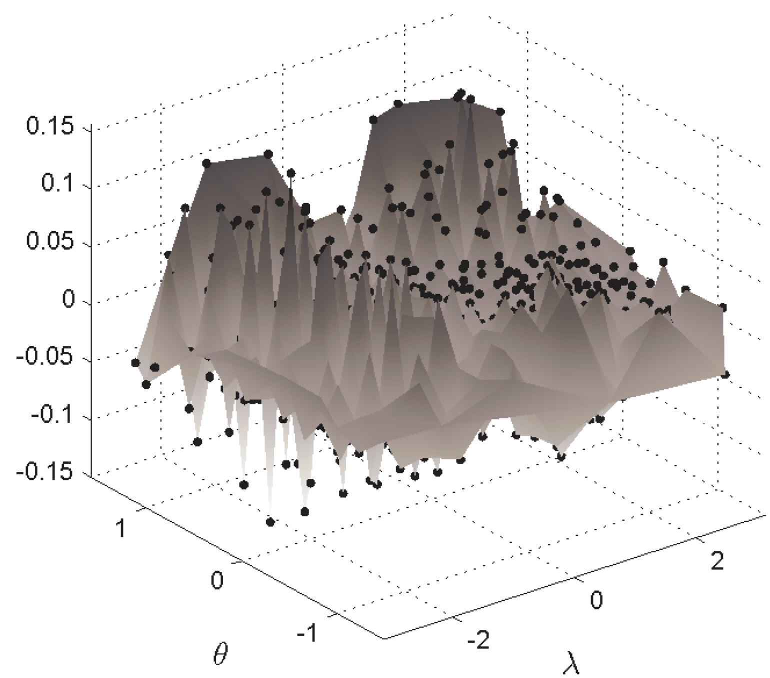
ε = 0.7



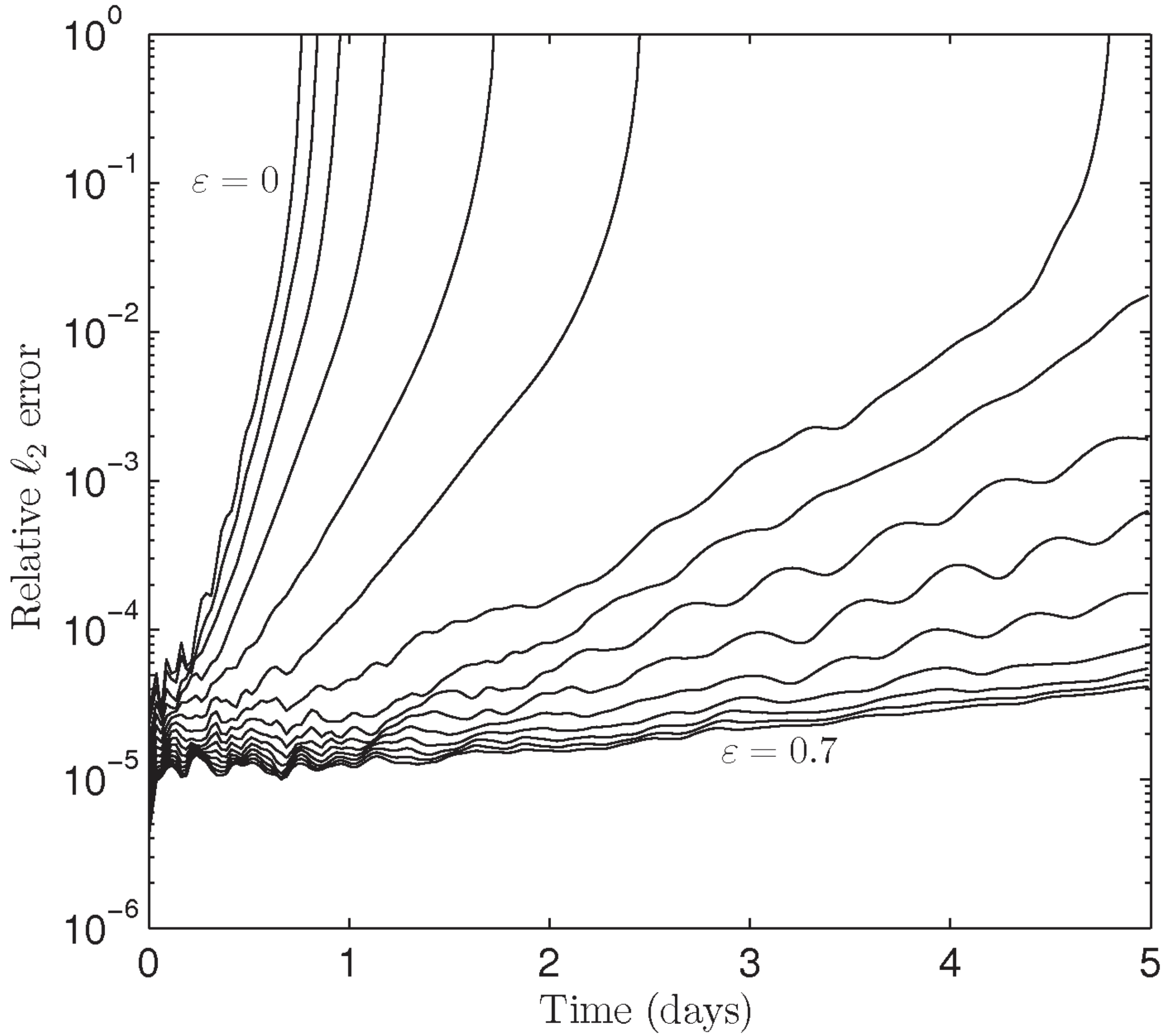
ε = 0.2



ε = 0.0



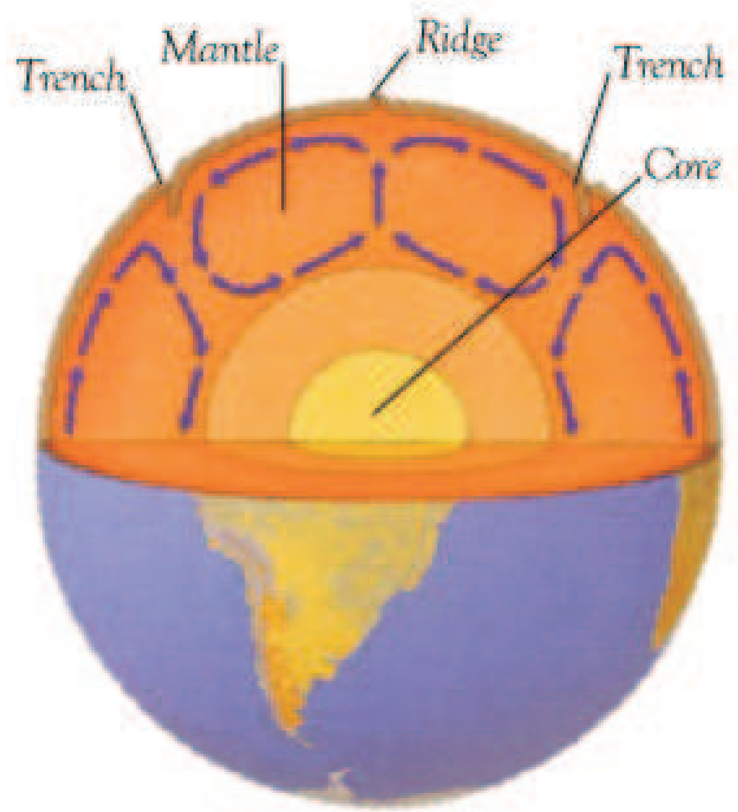
Error as a function of ε



$\varepsilon \to 0 \Rightarrow$ basis reproduces SH $\Rightarrow$ SH notorious for aliasing $\Rightarrow$ Filter always imposed

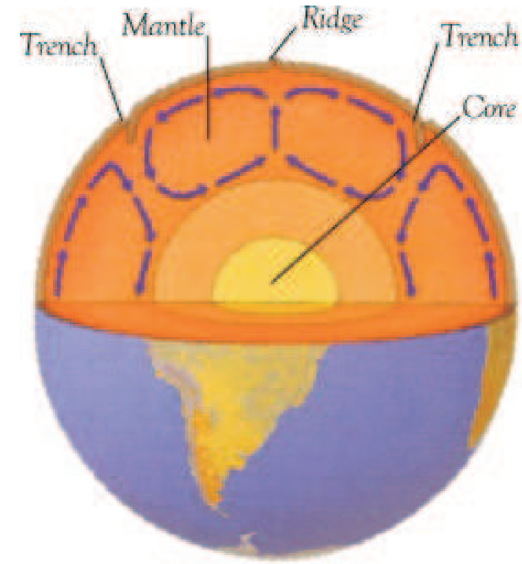
Next Step: Go Full Dynamic 3D in Spherical Geometry

Mantle (Thermal) Convection in a 3D Spherical Shell



The Physical Model

- Infinite Prandtl Number, $\mathrm{Pr} = \dfrac{\text{kinematic viscosity}}{\text{thermal diffusivity}} \to \infty$
- Constant Viscosity



Equations:

$$\begin{aligned} \nabla \cdot \underline{u} &= 0 && \text{Incompressible} \\ \nabla \cdot (\nabla \underline{u} + (\nabla \underline{u})^T) + \mathrm{Ra}\, T \hat{\boldsymbol{r}} &= \nabla p && \text{Stokes flow} \\ \frac{\partial T}{\partial t} + \underline{u} \cdot \nabla T &= \nabla^2 T && \text{Advection-Diffusion} \end{aligned}$$

Ra = Rayleigh number; related to ratio of heat convection to heat conduction

Momentum: $\underline{u} = \nabla \times \nabla \times \Phi \hat{\boldsymbol{r}}$ Φ = velocity potential (Chandrasekhar,1961)

$$\nabla^4 \Phi = \mathrm{Ra}\, T \longrightarrow \begin{array}{l} \triangle \Omega = \mathrm{Ra}\, T \\ \triangle \Phi = \Omega \end{array}$$

BCs: $\underline{u}$ is shear-stress free (slip) on impermeable inner and outer boundaries,

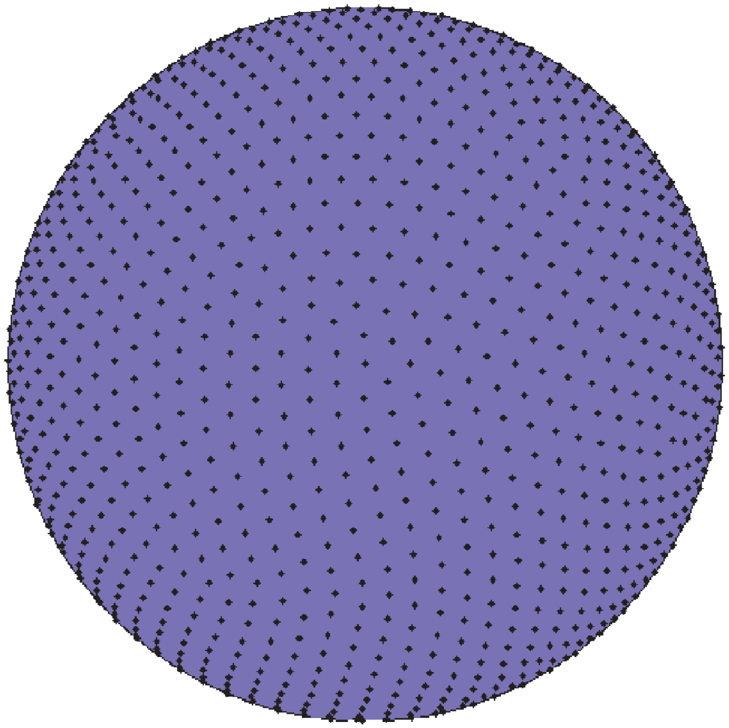
$T = 1$ at inner boundary (outer core of Earth)

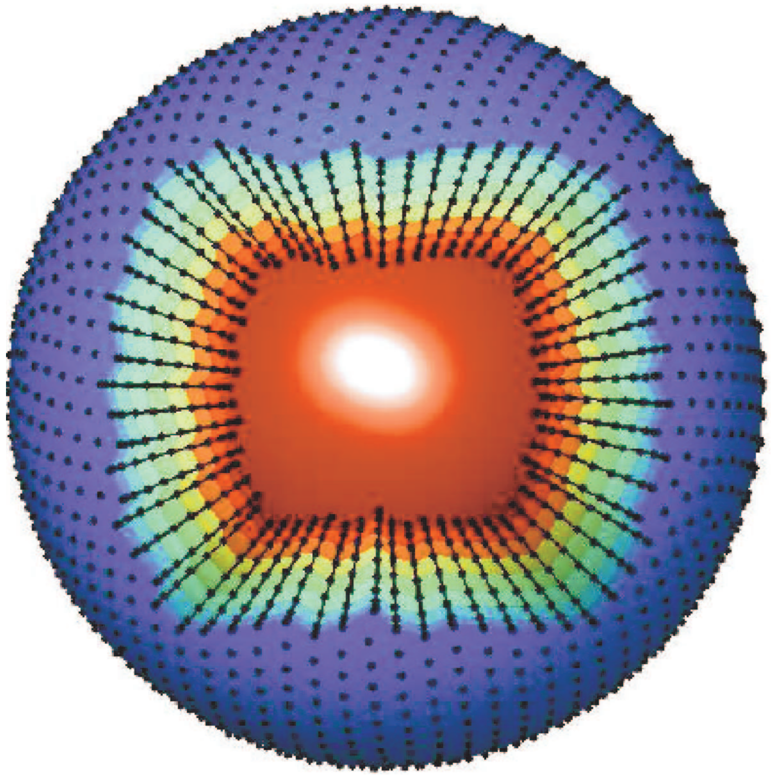
$T = 0$ at outer boundary (crust)

Hybrid RBF-Chebyshev

(Wright, Flyer, Yuen, *Geophysics, Geochemistry, Geosystems*, 2010)

Node Layout for hybrid RBF-Chebyshev discretization:





N **nodes on a shell - RBF**

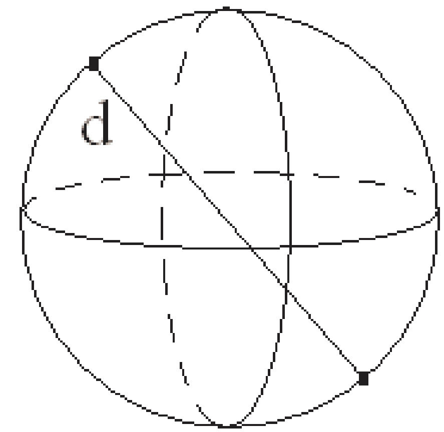
M **Chebyshev nodes radially**

RBF: Algorithmic Simplicity

(Wright, Flyer, Yuen, *Geochem., Geophy., Geosys.*, 2010)

Example:

$$d = \|\underline{x} - \underline{x}_k\| = \sqrt{(x-x_k)^2 + (y-y_k)^2 + (z-z_k)^2}$$



$$\Delta_{\boldsymbol{surface}}\, \phi(d) = \frac{1}{4}\left((4-d^{\mathbf{2}})\, \frac{\partial^2 \phi(d)}{\partial d^2} + \frac{4-3d^{\mathbf{2}}}{d}\, \frac{\partial \phi(d)}{\partial d} \right)$$

$\Delta_{\boldsymbol{surface}}\, \phi(d)$ Code using Gaussian RBFs, $e^{-(\boldsymbol{ep^2 d^2})}$, (**2 lines**)

```
d2      =  2 * (1 -  (x*x' + y*y' + z*z'))  ;

Lsfc    = 1/4 *( (4 - d2)* (-2*ep^2 *exp(-ep^2*d2) + 2*ep^4 *d2*exp(-ep^2*d2)) +
                (4 - 3*d2)./sqrt(d2)* (-2*ep^2*sqrt(d)*exp(-ep^2*d2)) );
```

Algorithmic Simplicity:

- *Independent of Dimension*
- *Independent of Coordinate System*

RBF Computational Algorithm

(i) $\nabla^4 \Phi = Ra\, T \Leftrightarrow \begin{cases} \Delta_{surface}\, \Omega + \dfrac{\partial}{\partial r}\left(r^2 \dfrac{\partial}{\partial r}\right)\Omega = r^2 \mathrm{Ra} T \\ \Delta_{surface}\, \Phi + \dfrac{\partial}{\partial r}\left(r^2 \dfrac{\partial}{\partial r}\right)\Phi = r^2 \Omega \end{cases}$

(ii) $\underline{u} = \nabla \times \nabla \times \Phi \hat{\boldsymbol{r}}$

(iii) $\dfrac{\partial T}{\partial t} + \left[\underline{u} \cdot \left(\nabla_{surface} + \dfrac{\partial}{\partial r}\boldsymbol{e}_r\right)\right] T = \left[\Delta_{surface} + \dfrac{\partial}{\partial r}\left(r^2 \dfrac{\partial}{\partial r}\right)\right] r^2 T$

1) Discretize $\Delta_{surface}$, $\dfrac{\partial}{\partial \theta}$, $\dfrac{\partial}{\partial \varphi}$ using N RBFs

2) Discretize $\dfrac{\partial}{\partial r}$, $\dfrac{\partial^2}{\partial r^2}$ using M Chebyshev polynomials

3) Use T initial condition to solve for Ω ↘

Use eigenvector decomposition

4) Use Ω solution to solve for Φ ↗ $O(N^2 M)$ instead of $O(N^2 M^2)$

5) Use Φ solution to calculate $\underline{u}$

6) Discretize time using a *time-splitting* scheme

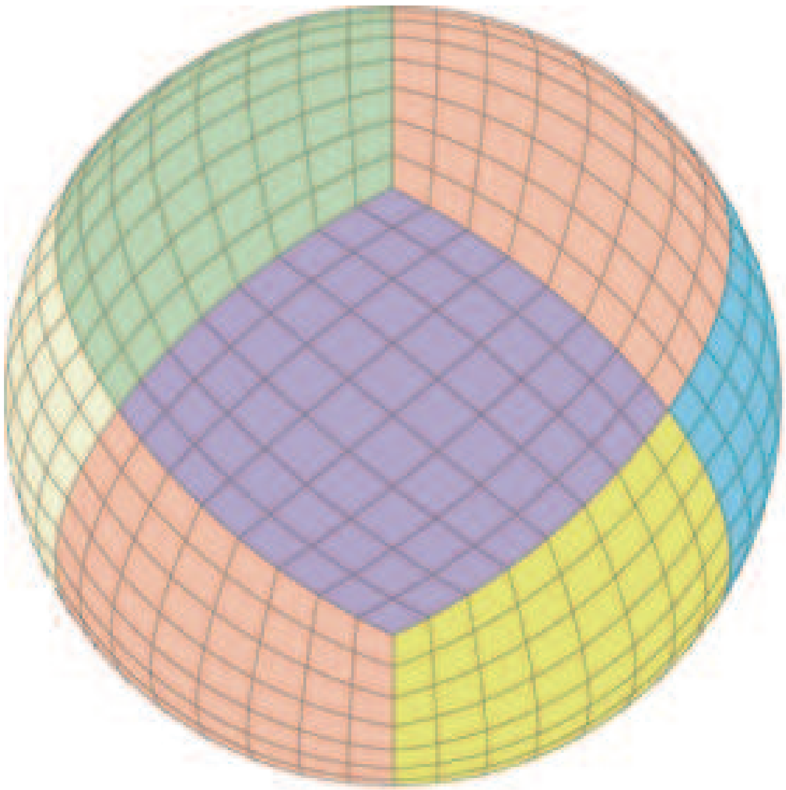
- 2nd order Adams-Moulton (AM2) for diffusion operator (implicit)
- 3rd order Adams-Bashforth (AB3) for advection operator (explicit)

7) Time-step energy equation to get new field for T, Back to Step 3

Comparative Study

Three Numerical Methods

1) RBF - Chebyshev
2) Finite Element (NSF funded - CitcomS)
3) Spherical Harmonic - Finite Volume or Finite Difference (CNRS - France, Germany)



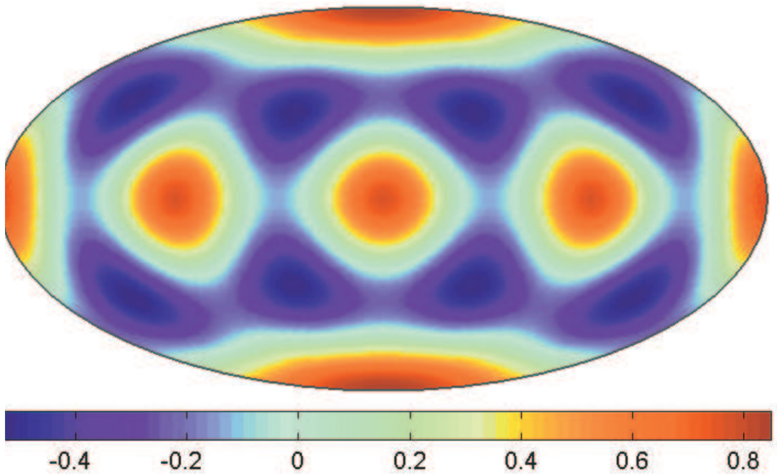
CitcomS

12 Equal Caps

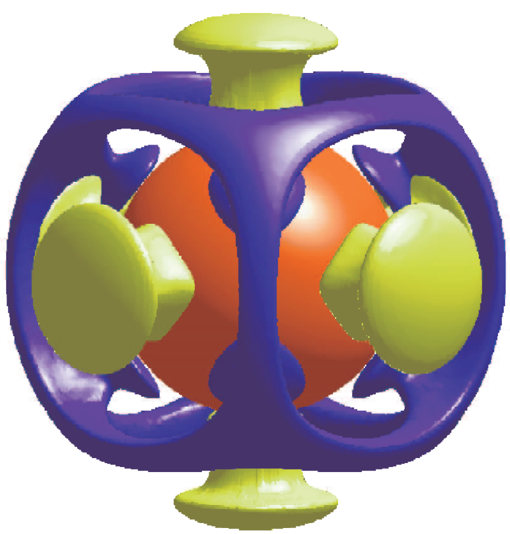
Each *N* x *N* x *N* Elements

Second - Order

Community Benchmark, *Ra* = 7,000: Validation of RBF Method



Initial condition: $\underbrace{f(r)}_{\text{sinusoidal in } r}\left[Y_4^0 + \frac{5}{7}Y_4^4\right]$



Isosurface of perturbation temperature
Blue: down-welling, Yellow: up-welling, Red: core

N =1600 nodes on each spherical shell
M = 23 shells

Total: 36,800 unknowns
8 min. 16 secs wall-clock time
(desktop with single quare core processor)

Comparisons against main previous results from the literature

Nu = ratio of convective to conductive heat transfer across a boundary

For Steady State with No Internal Heating $\Longrightarrow$ $Nu_{outer} = Nu_{inner}$

Method	No of nodes	Nu_{outer}	Nu_{inner}	$\langle V_{RMS} \rangle$	$\langle T \rangle$
Finite volume	663,552	3.5983	3.5984	31.0226	0.21594
Finite elements (CitcomS)	393,216	3.6254	3.6016	31.09	0.2176
Finite differences (Japan Earth Simulator)	12,582,912	3.6083		31.0741	0.21639
Spherical harmonics -FD	552,960	3.6086		31.0765	0.21582
Spherical harmonics -FD	Extrapolated	3.6096		31.0821	0.21577
RBF-Chebyshev	36,800	3.6096	3.6096	31.0820	0.21578

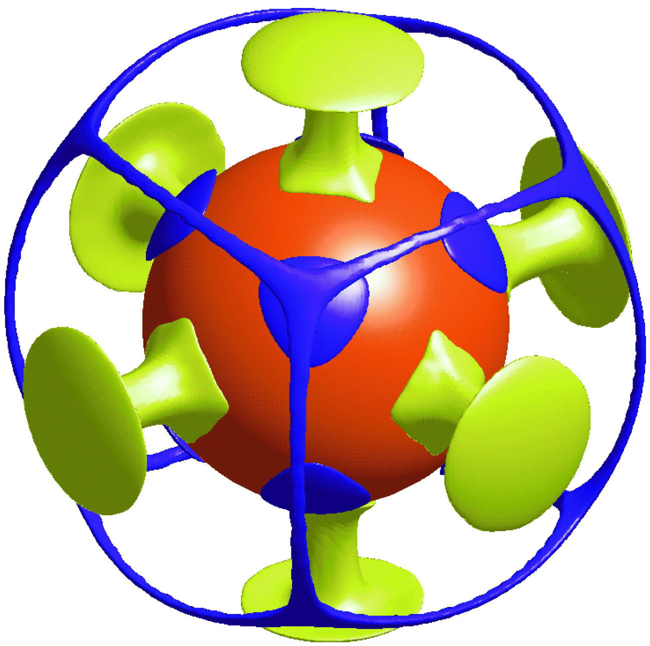
RBFs Nail The Answer!

And Higher *Ra*....?

Traditional view: Unsteady flow does not occur till $Ra \sim O(10^5)$

Lower *Ra* is uninteresting $\rightarrow$ goes to stable steady state

Reasonable Assumption? Yes



Method	No. of nodes	Wall Clock Time
Finite elements (CitcomS)	1,411,788	2.78 days (12)
Spherical harmonics -FV	1,638,400	$\sim$ 2 days (8)
RBF-Chebyshev	176,128	6 hours 27 mins (1)

RBF-CH only used 1 CPU, others used multiple processors

Fast turn-around gave us the ability to question the physics

***Ra* = 100,000**

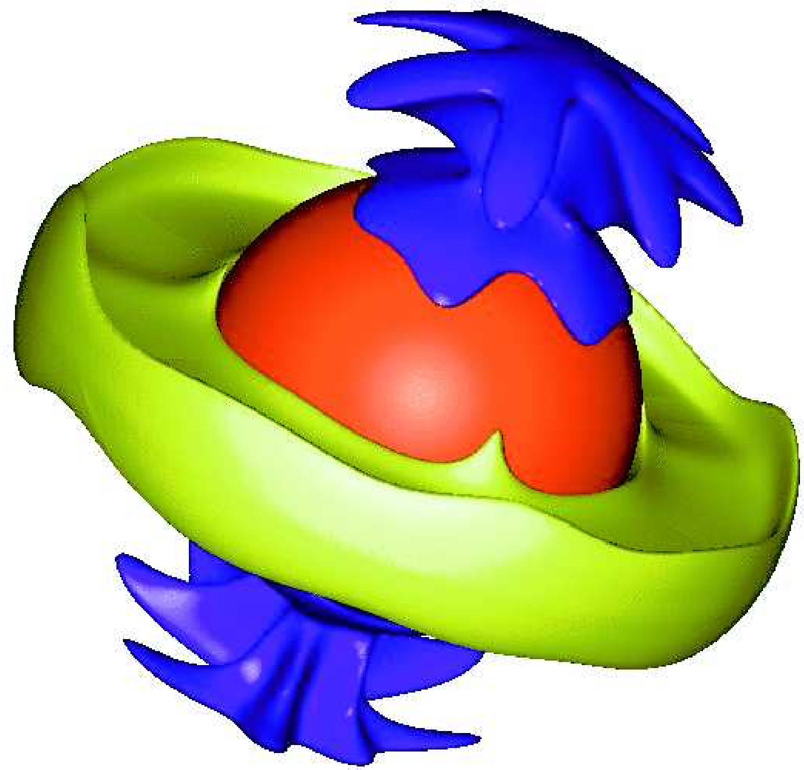
Isosurface of perturbation temperature

Blue: down-welling, Yellow: up-welling, Red: core

Same Ra but higher perturbation regime

At t = 0.236 or 13.5 billion years

RBF-Chebyshev



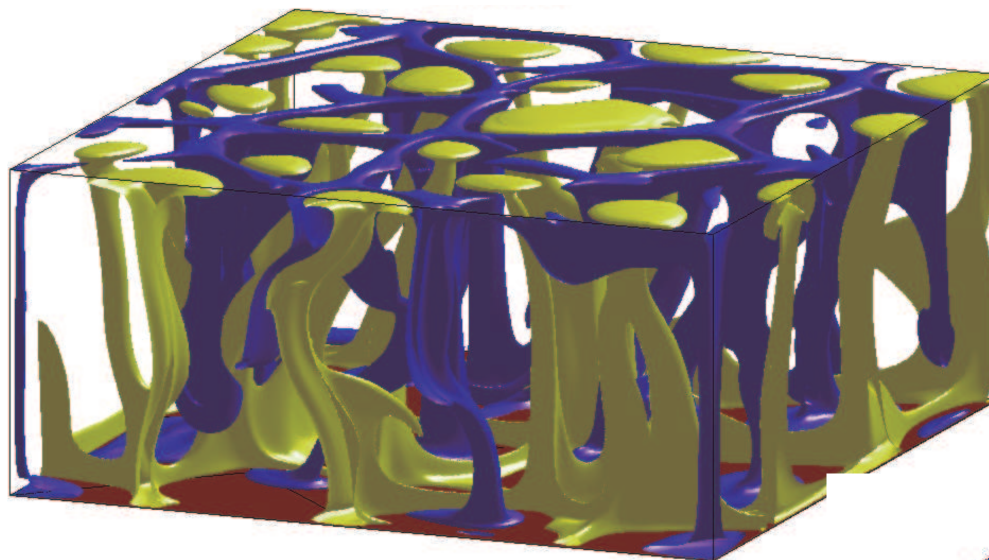
CitcomS

High Ra: Comparing Two Novel Simulations

Novelty: First mantle convection model run on a Graphics Processing Unit (GPU)
Strength: Speedup times up to a factor of 15
Drawback: 2nd-order, very dissipative, non-spherical geometry

Nodes: 32 million
Time step: 34,000 yrs

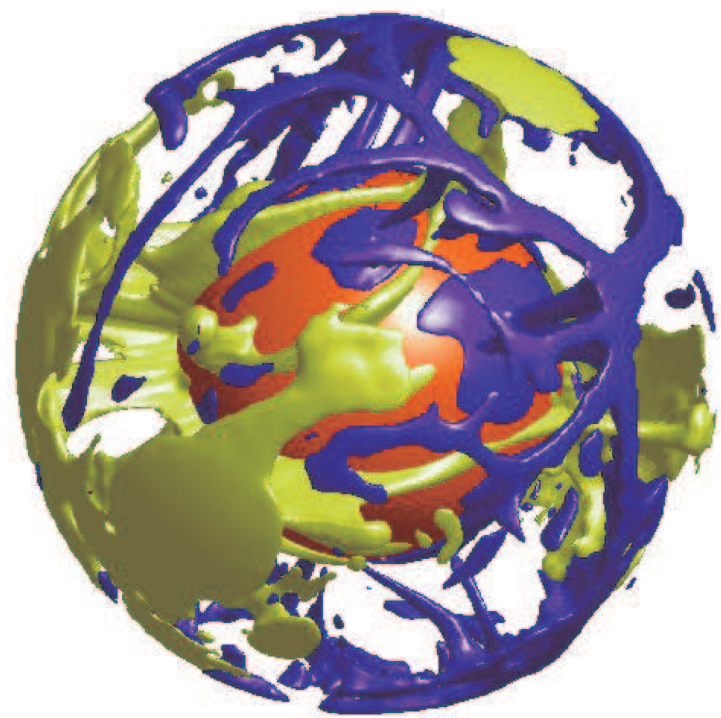
$\boxed{\mathbf{Ra} = \mathbf{10^7}}$



Novelty: Largest RBF simulation
Strength: Only Spectral accurate simulation
Drawback: Computationally slow

Nodes: 531,411
Time step: 34,000 yrs

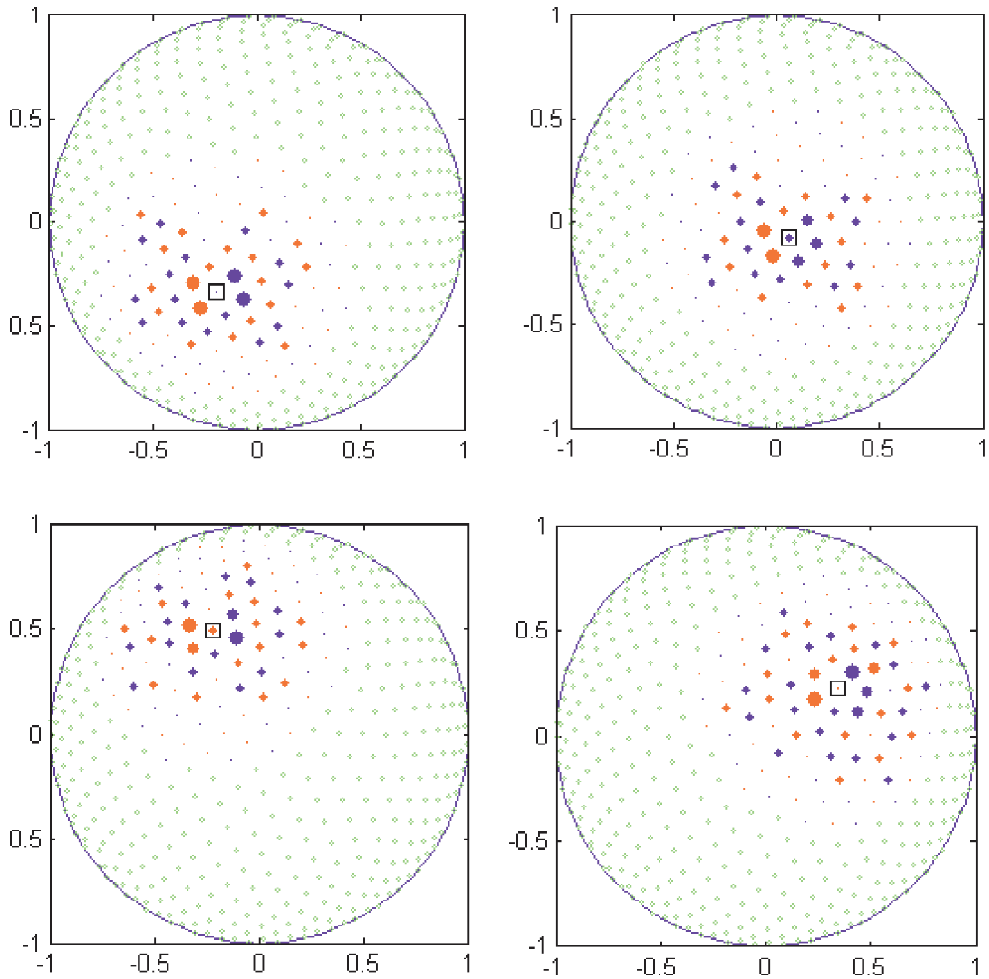
$\boxed{\mathbf{Ra} = \mathbf{10^6}}$



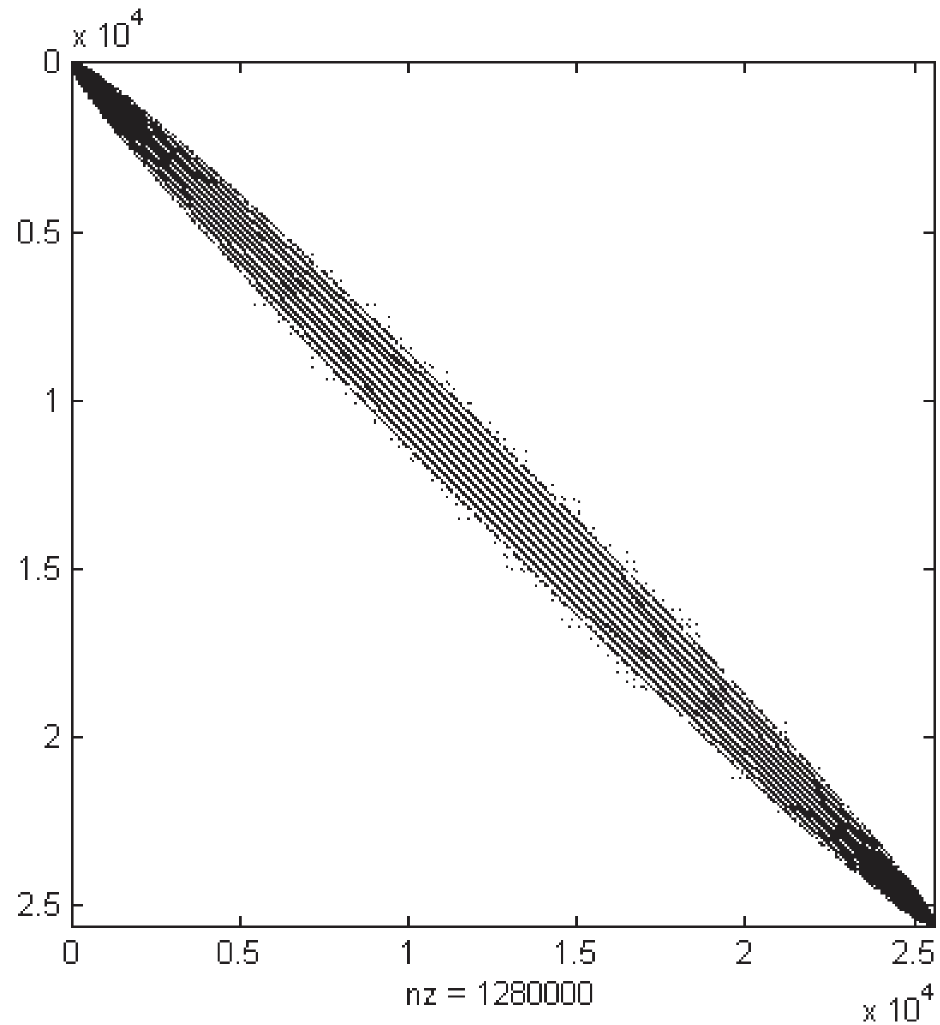
Reducing computational cost: RBF-based Finite Differences

Calculate RBF derivative approximations using a localized stencil rather than all nodes

4 stencils to calculate derivative approximations at square marker using 75 nearest nodes



RBF-FD matrix to approximate d/dx at all nodes for N=25000 , local stencil size=50



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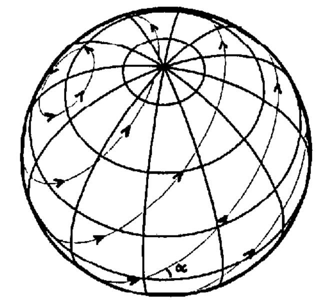
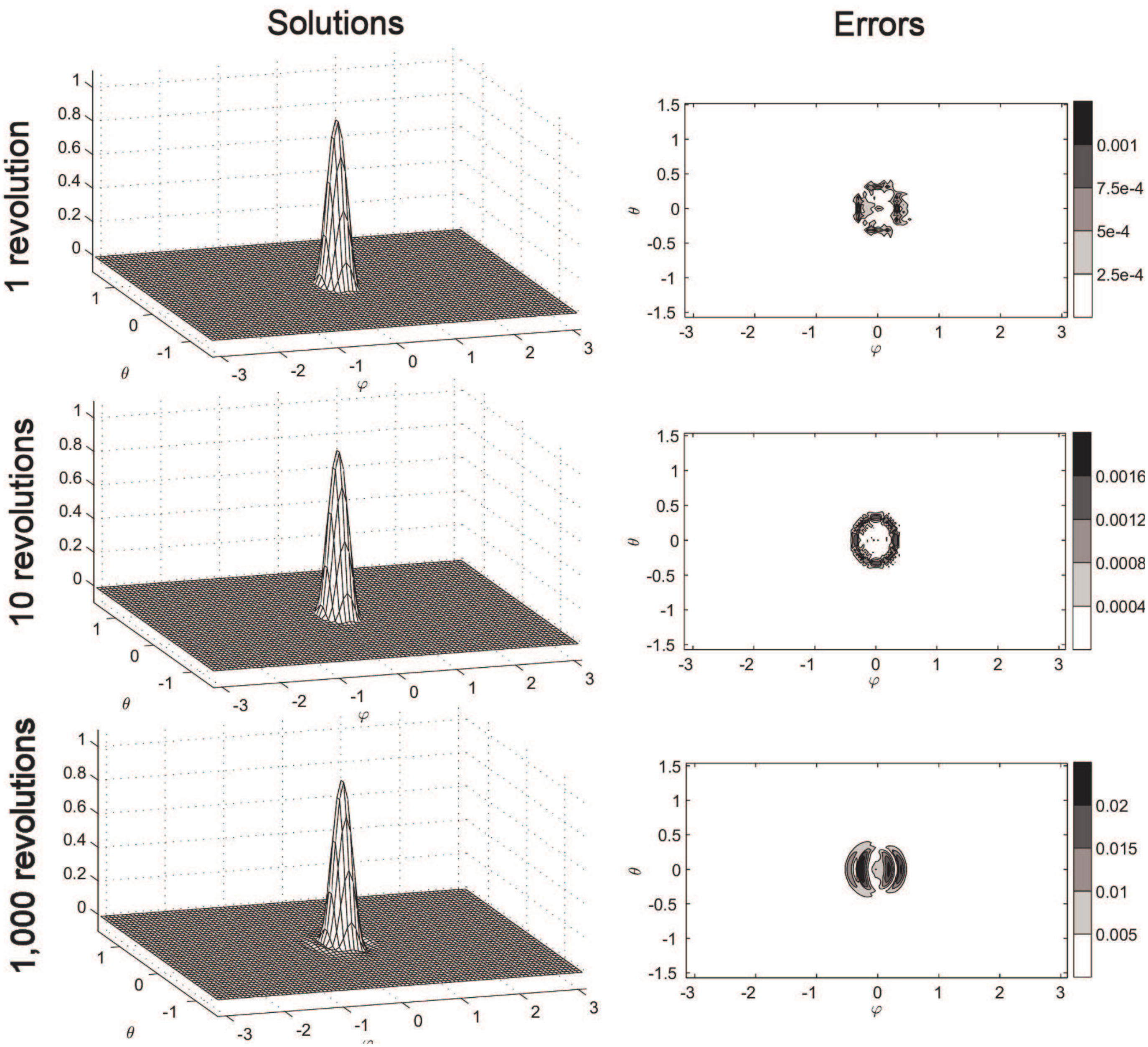
Solid Body Rotation of C^1 Cosine Bell with RBF-FD

(Fornberg and Lehto, JCP, 2011)

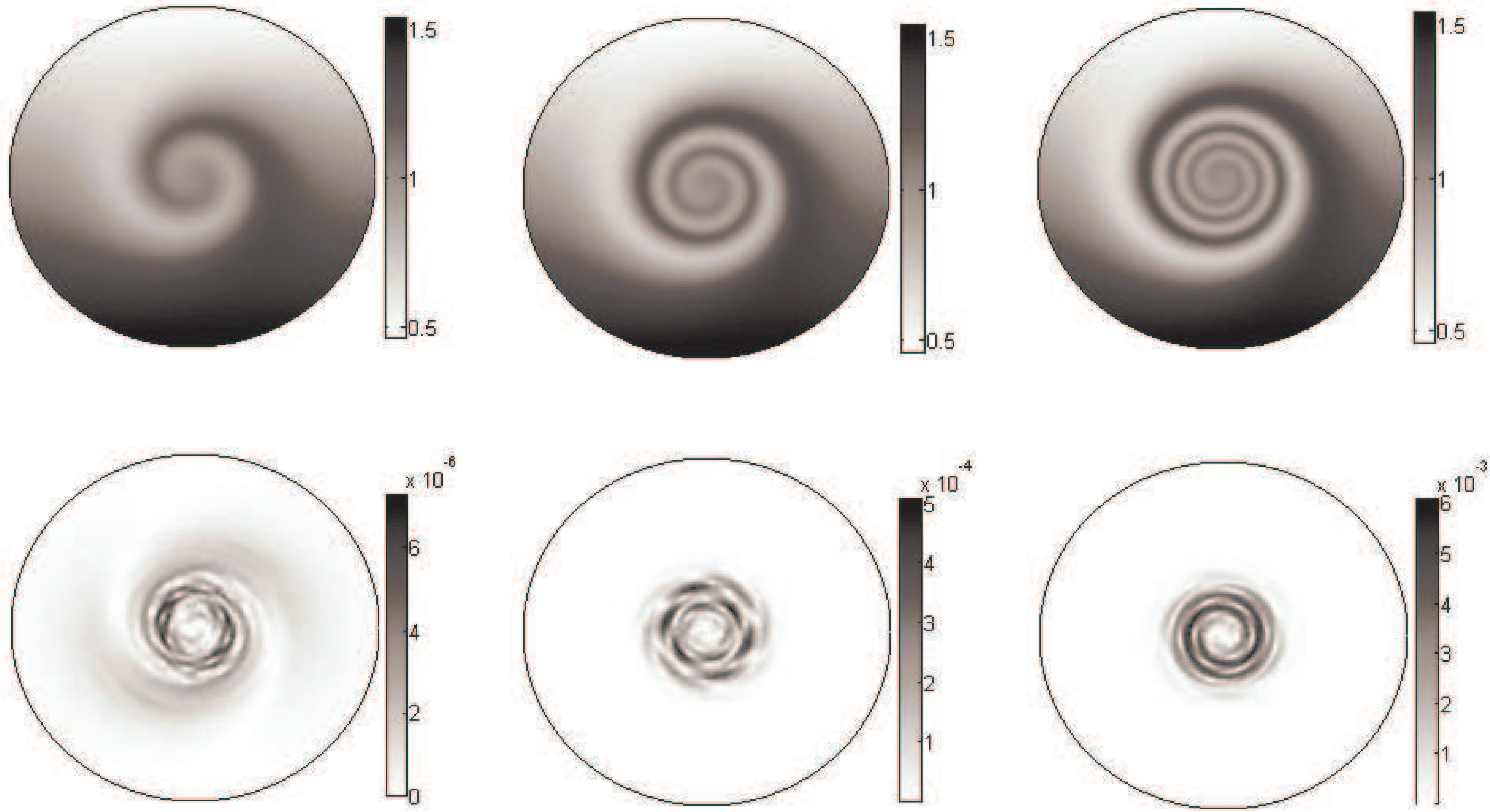




N = 25,000
local stencil size = 74
Δ^8 - type hyperviscosity

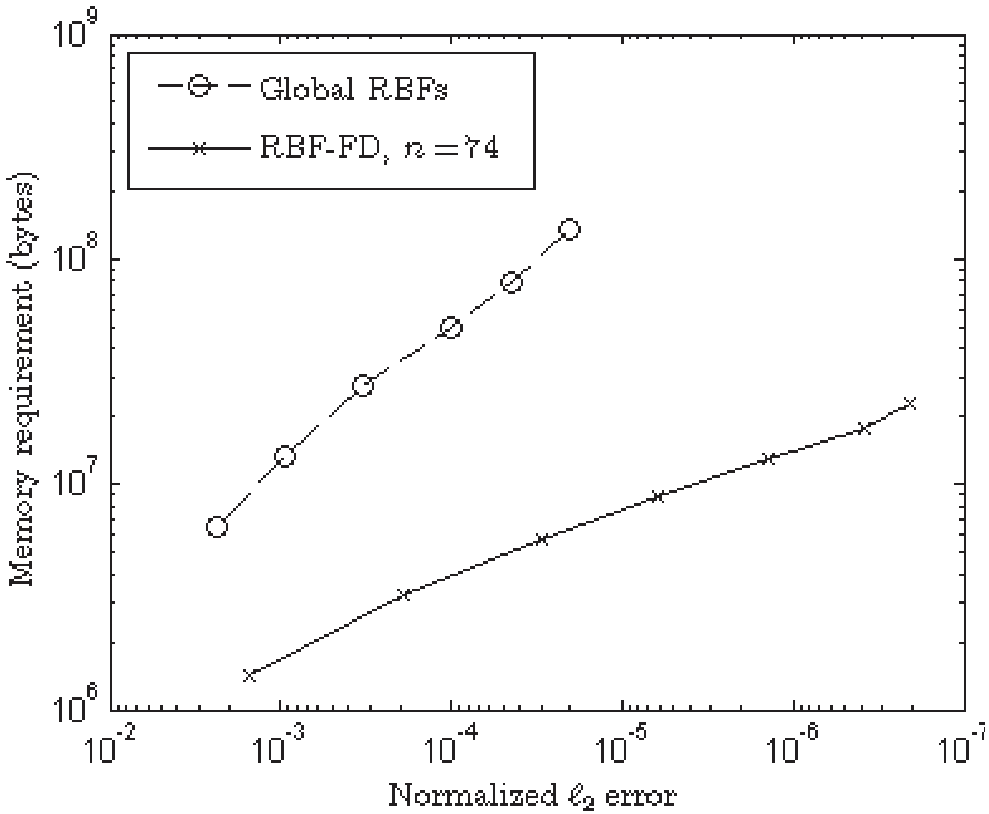
Stationary Vortex Wrap-up with RBF-FD

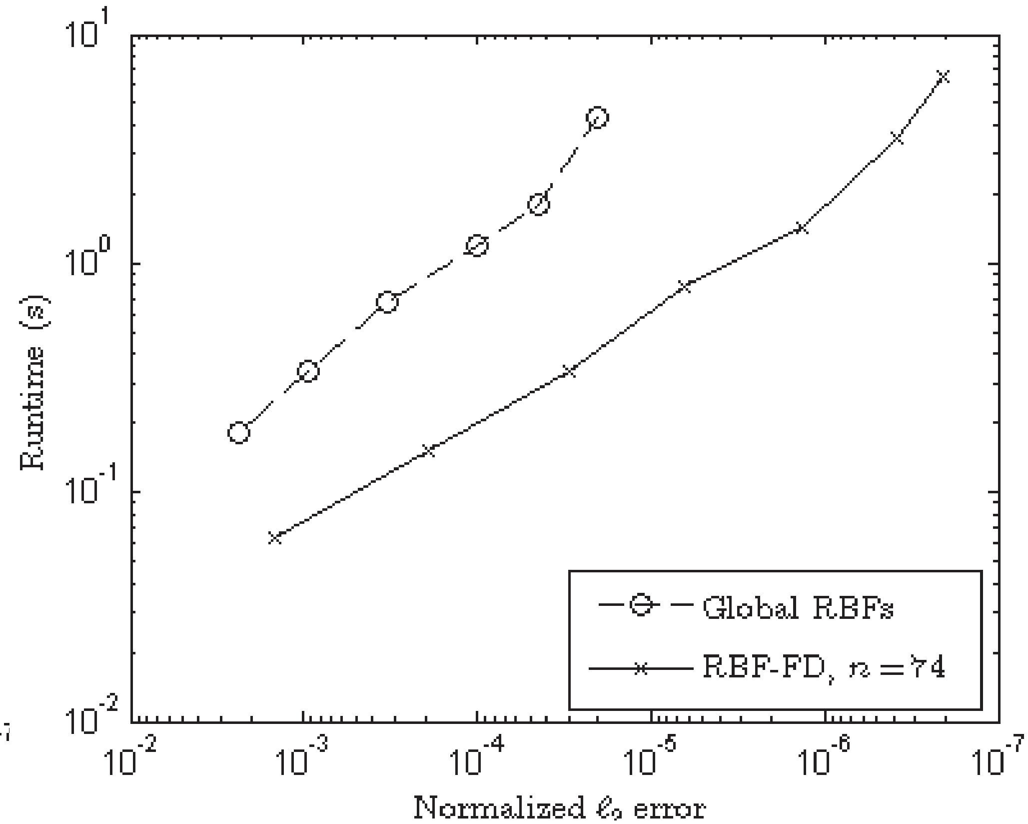
Total Number of nodes: 25,000 , Local stencil size: 50 , $\Delta t = 45$ minutes



Error: $O(10^{-6})$ $O(10^{-4})$ $O(10^{-3})$

A Look at Computational Cost Reduction (Vortex Wrap-up Case)

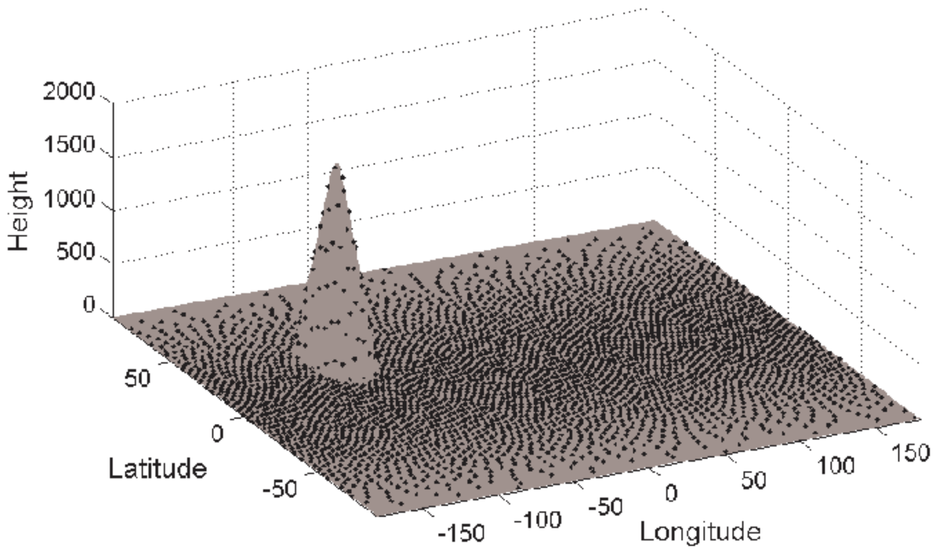




ℓ_2 Error 10^{-5}: 100MB (Global RBF)
7MB (RBF-FD)

ℓ_2 Error 10^{-5}: Runtime 10 seconds (Global RBF)
Runtime 0.7 seconds (RBF-FD)

Flow over an Isolated C^0 Cone Mountain with RBF-FD

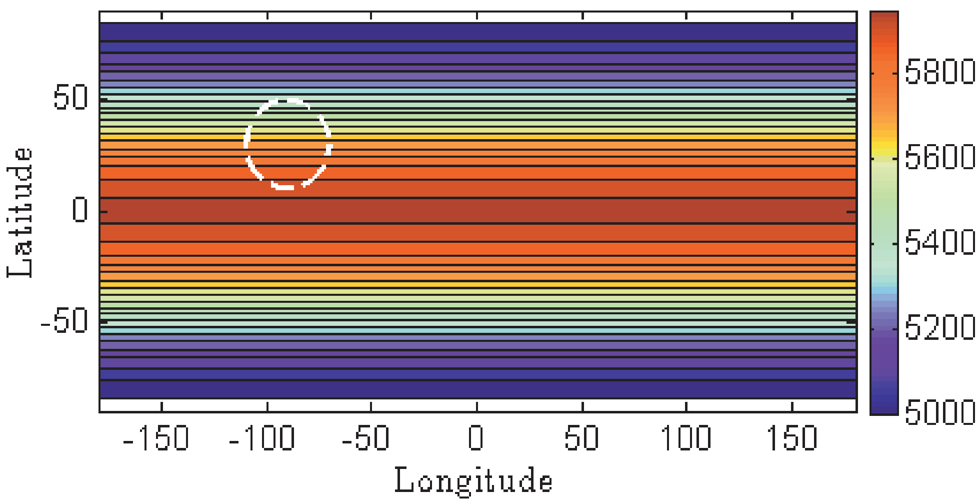


Shallow Water Equations
Look at:

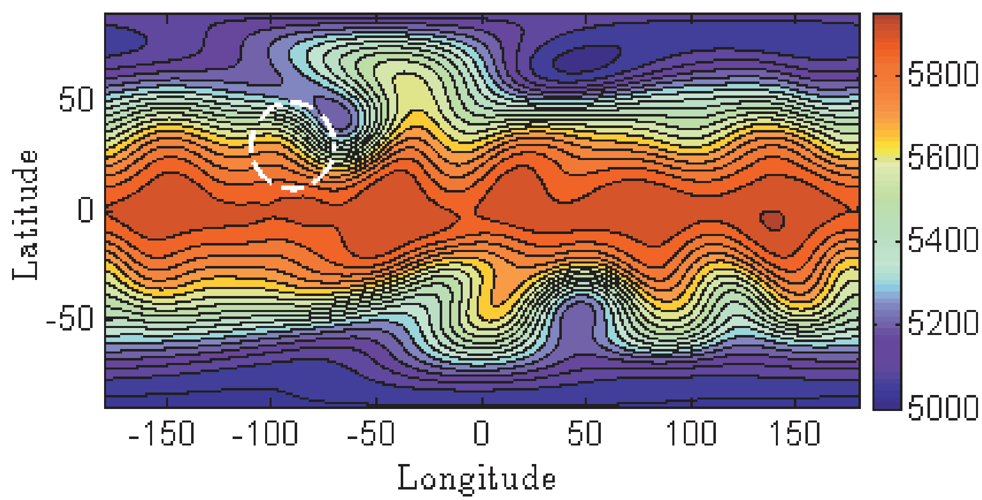
- Convergence
- Effects of Gibbs phenomena
- Hyperviscosity
- Runtime

Cone Mountain

N = 25,600 stencil size = 31



h, t = 0 days



h, t = 15 days

Specifications for RBF-FD Runs for C^0 Cone Mountain

N	Stencil size, n	Resolution (km)	Time step (min)	ℓ_2
3,600	31	400	20	$5 \cdot 10^{-4}$
6,400	31	300	15	$2 \cdot 10^{-4}$
12,100	31	220	12	$1 \cdot 10^{-4}$
25,600	31	150	5	$6 \cdot 10^{-5}$
40,962	31	120	3	$2 \cdot 10^{-5}$
163,842	31	60	1	$1 \cdot 10^{-5}$

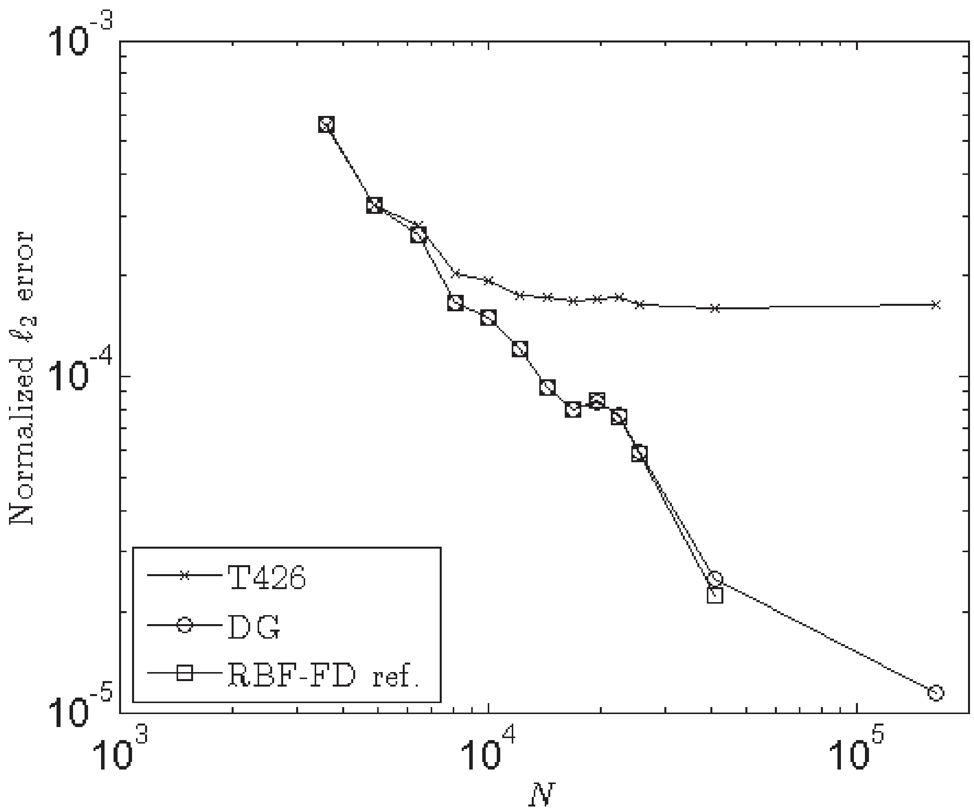
Less than 50 km resolution $\longrightarrow$ saturation error, stable algorithms needed

Convergence Comparison Using 3 Different Reference Solutions

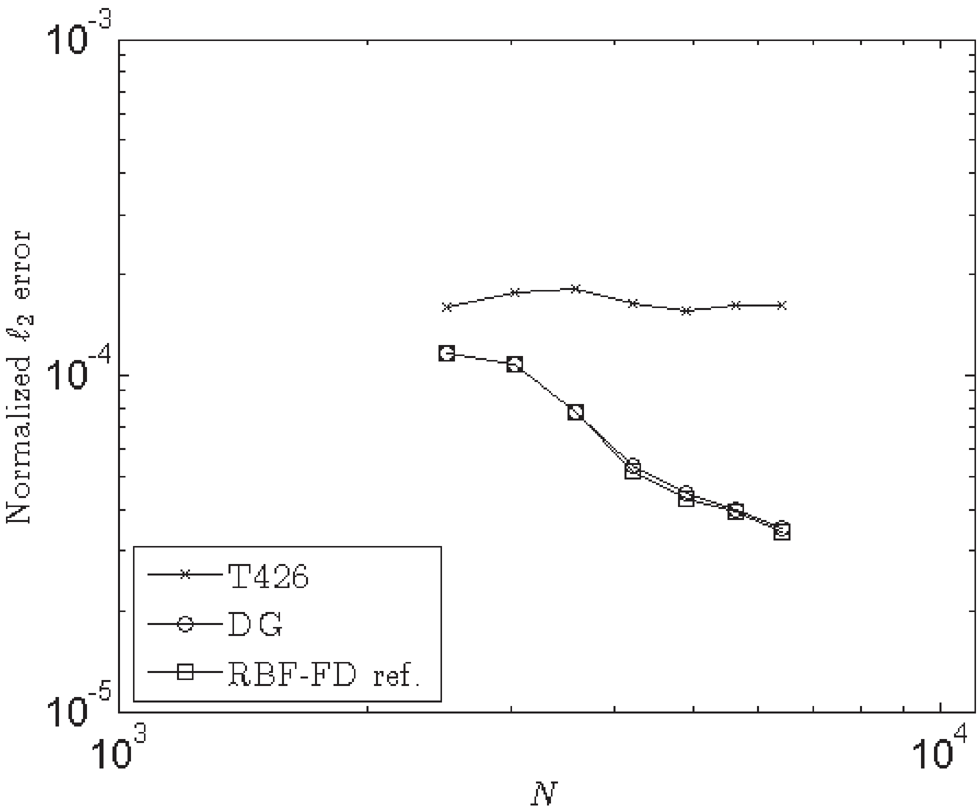
1) Standard of the Literature/Comparison: NCAR's Sph. Har. T426 $\approx$ 30 km at equator

2) New Model at NCAR Discontinuous Galerkin - Spectral Element, $\approx$ 30 km

3) RBF - FD, N = 163,842, $\approx$ 60 km



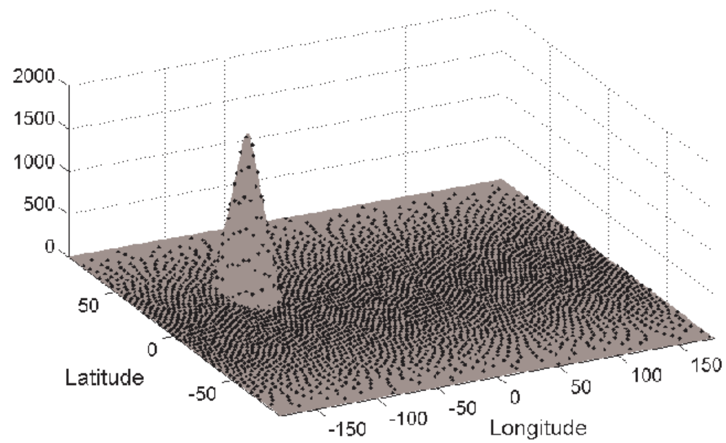
RBF-FD, Stencil Size: n = 31



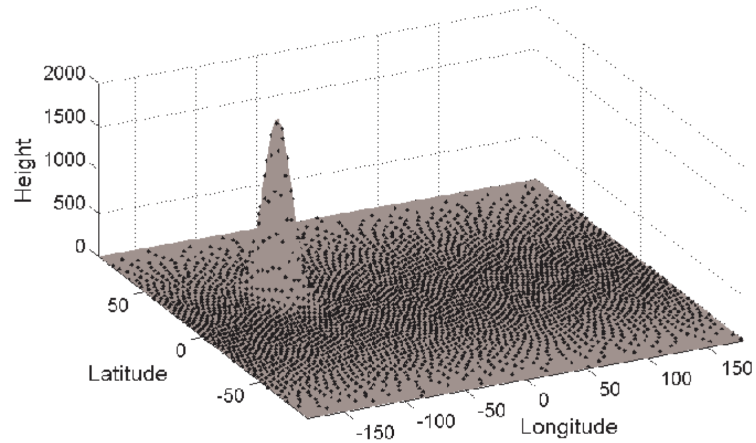
Global RBF

First evidence that the standard of comparison for over a decade,
NCAR's Spectral Transform Model, is not so accurate.

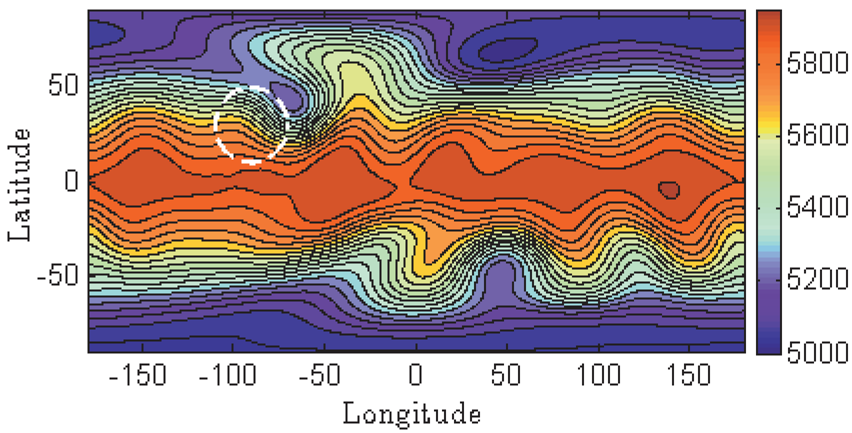
Error due to Gibbs Phenomena (Reference solution is 30km DG - SE)

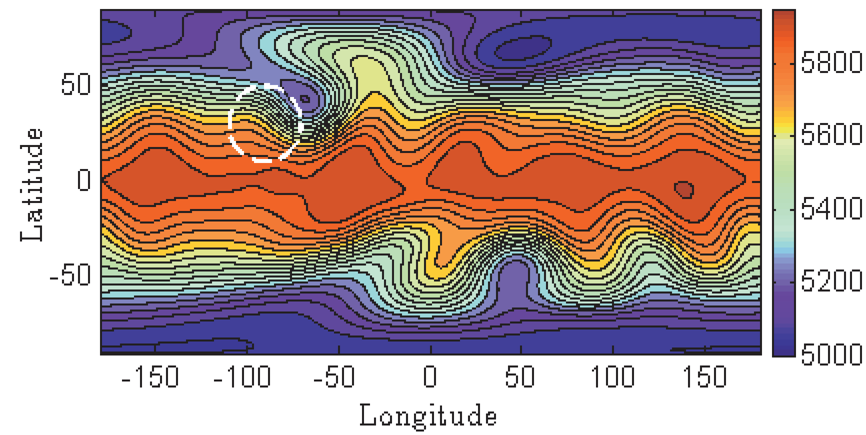


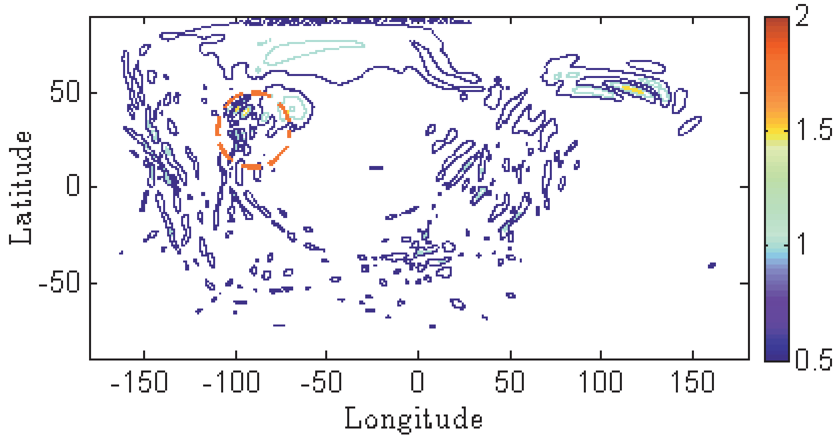
Cone

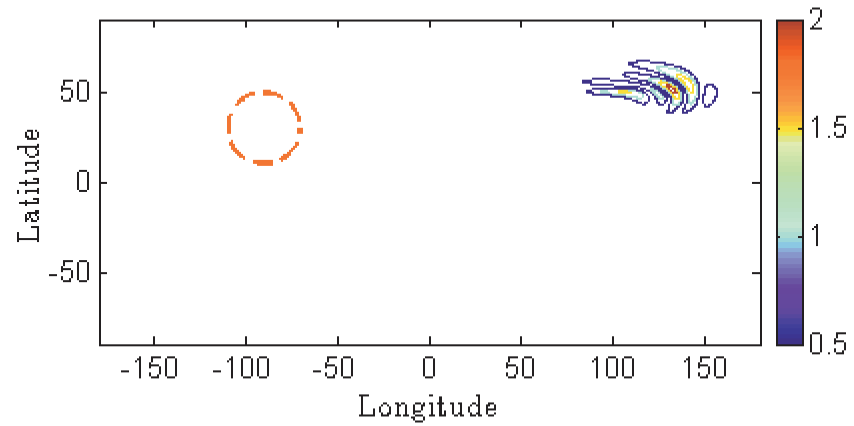


GA



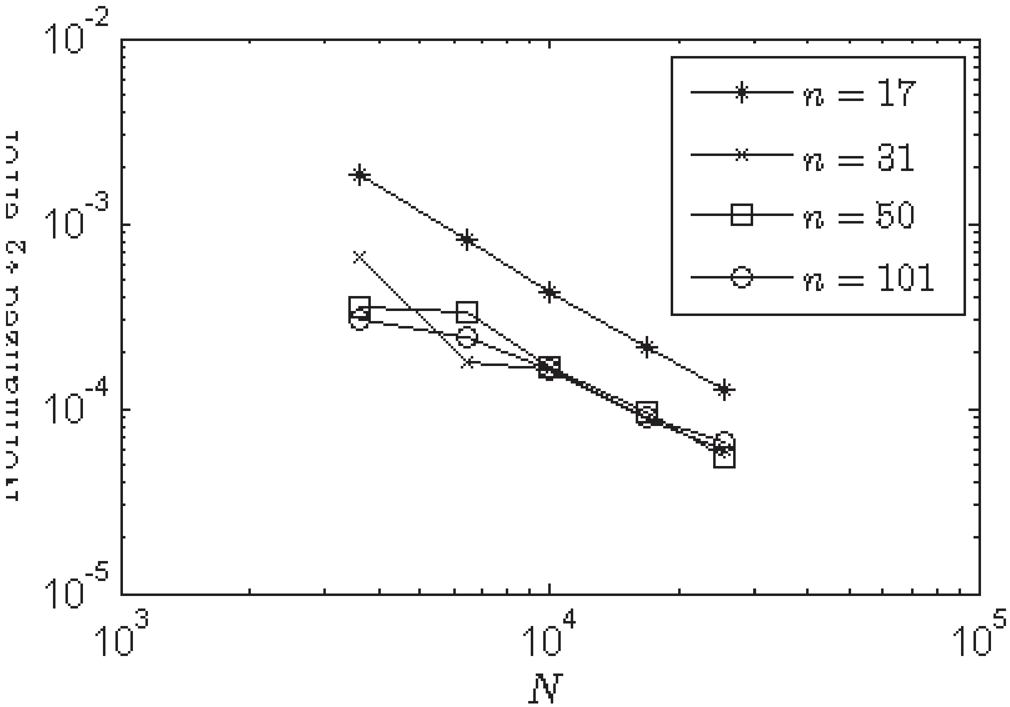




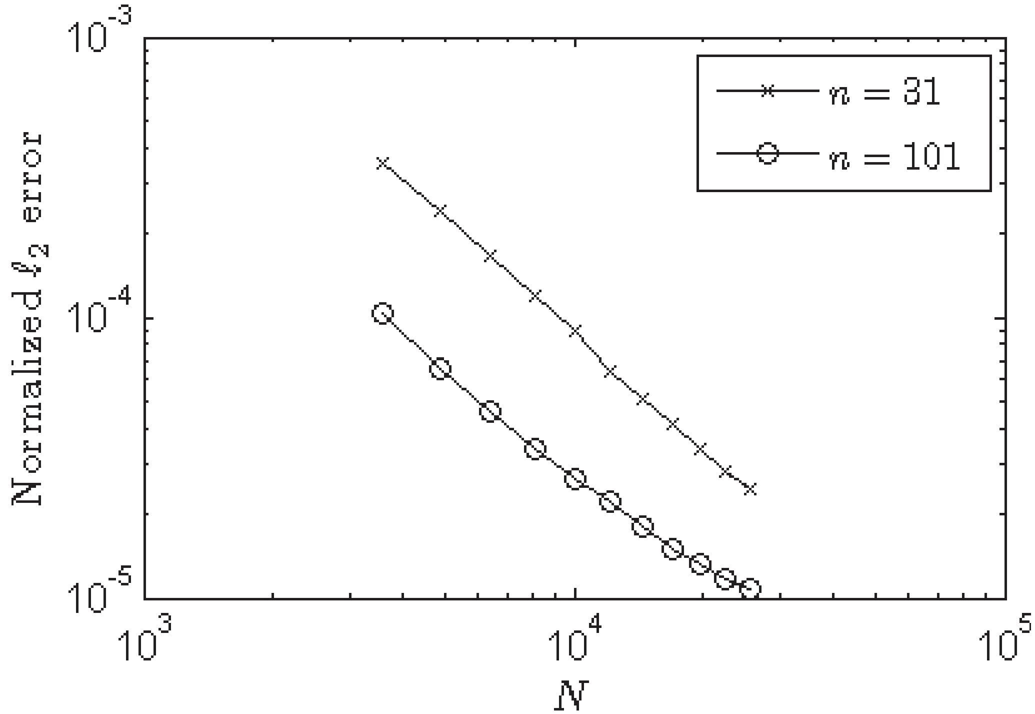


Effect of Gibbs on Stencil Size

Reference solution is 60 km RBF-FD.



Cone Mountain



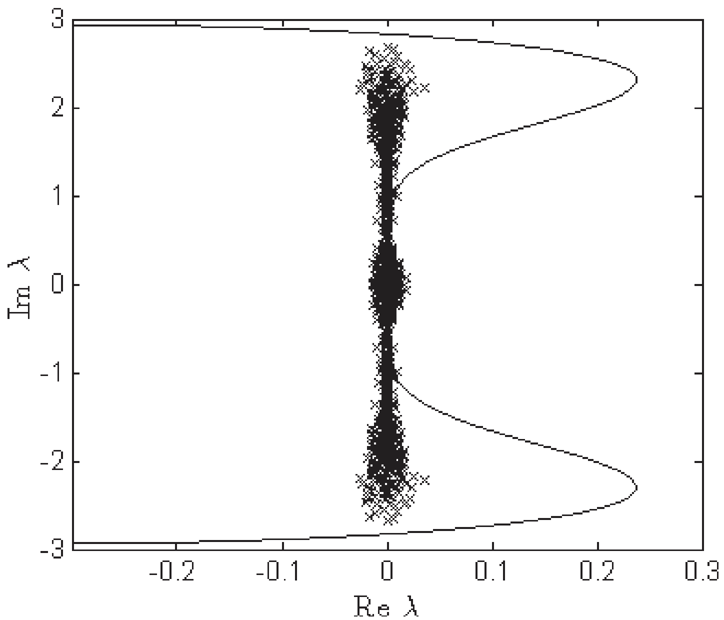
Gaussian Mountain

As stencil size increases for fixed *N*, derivative approximations become more global but accuracy is not increased due to non-smooth forcing

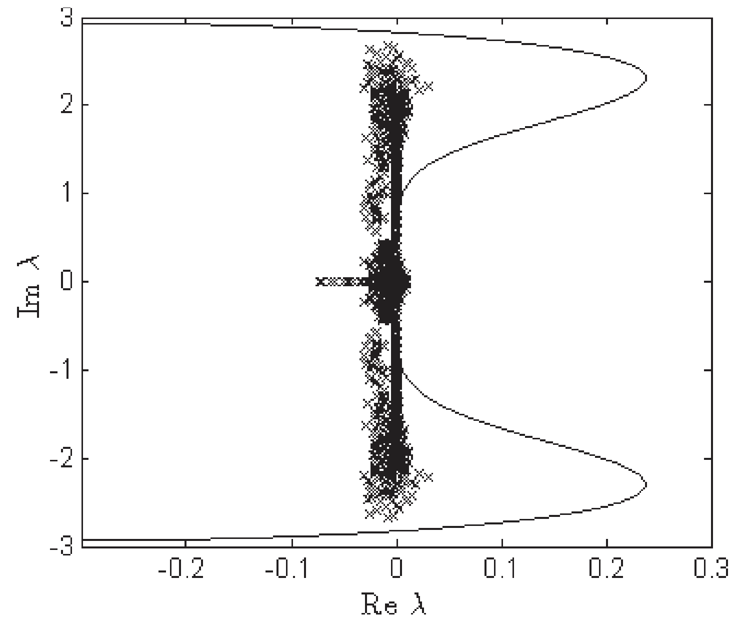
For smooth forcing, accuracy increases with stencil size but rate of convergence is not much greater due to steepness of mountain

Hyperviscosity

No hyperviscosity

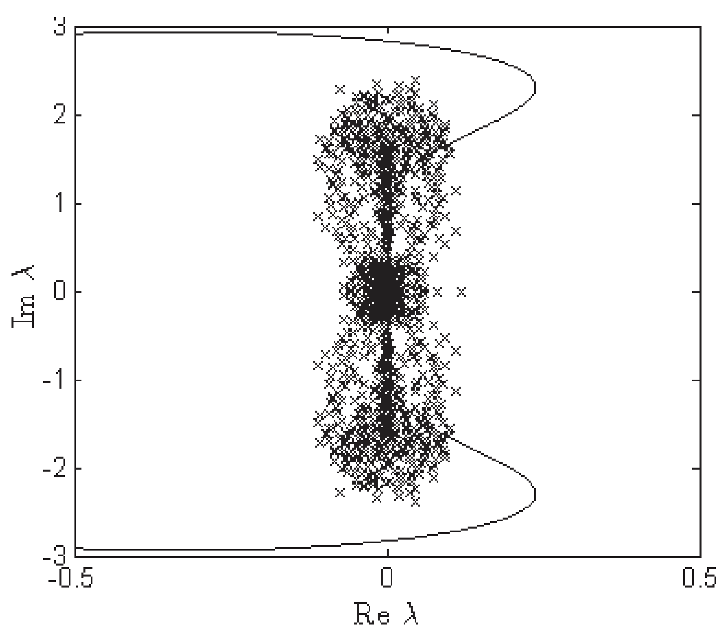


Hyperviscosity A^{-1} - type

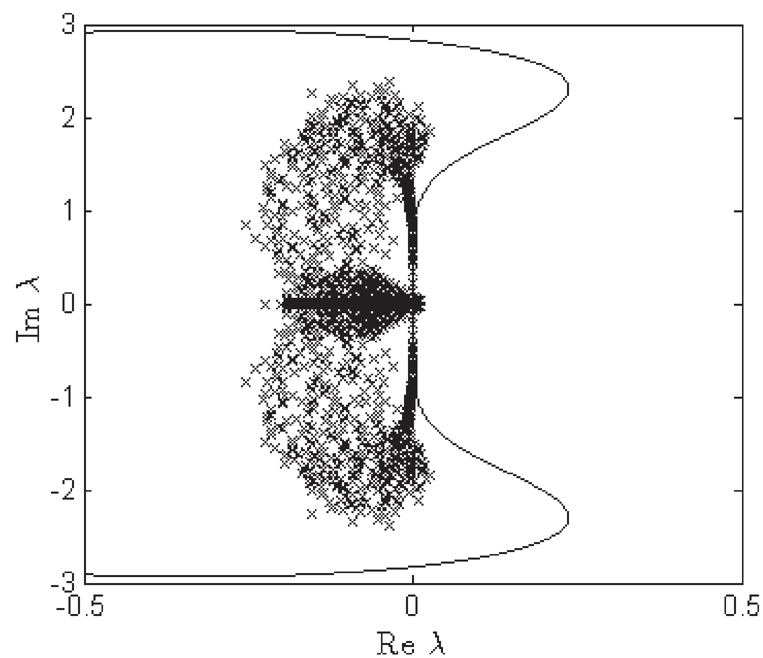


Global

No hyperviscosity



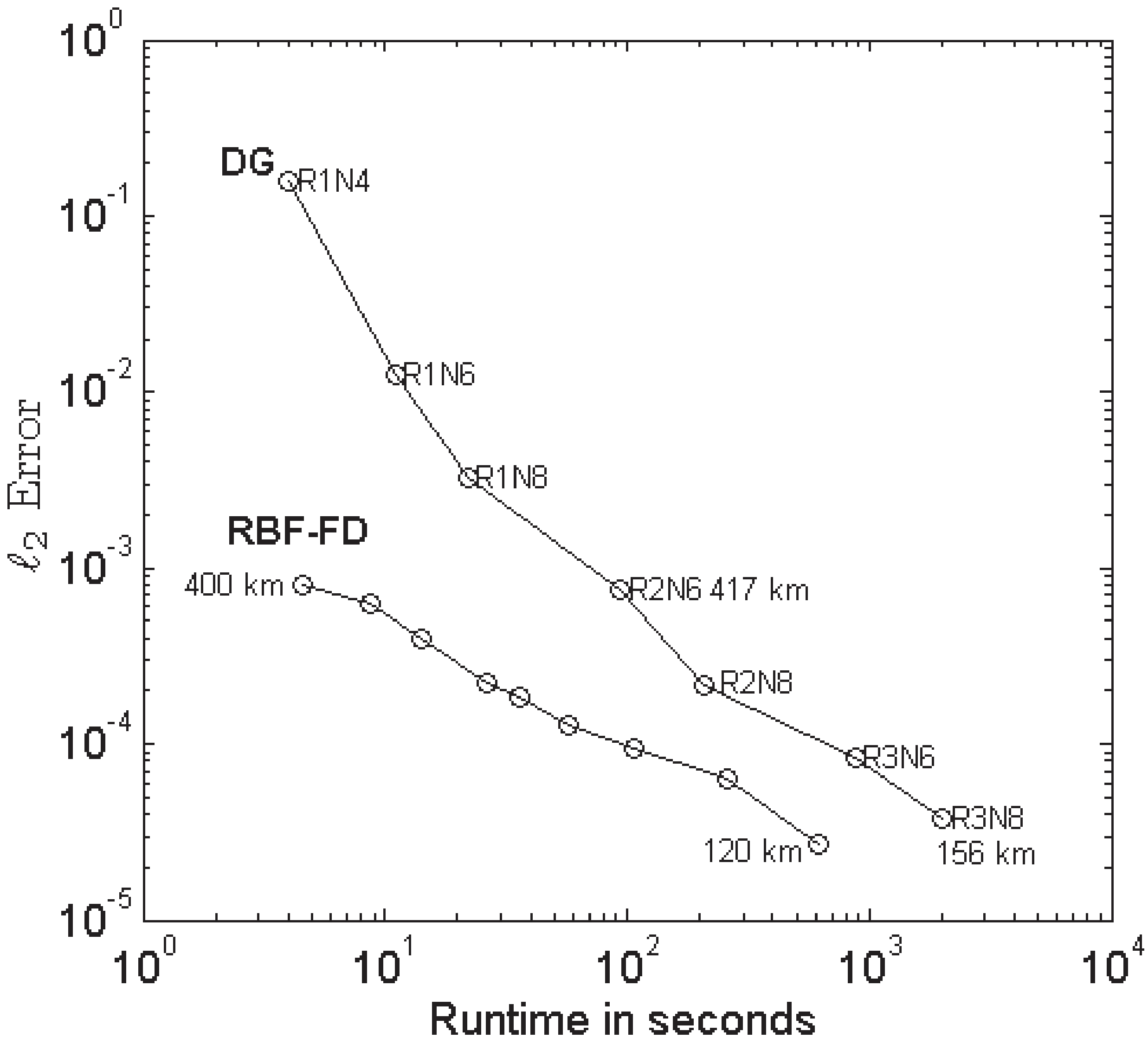
Hyperviscosity Δ^4 - type

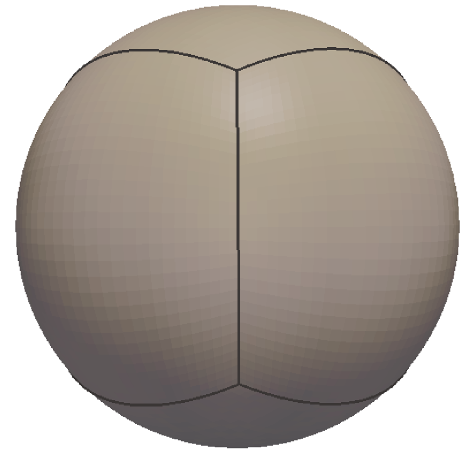


RBF-FD

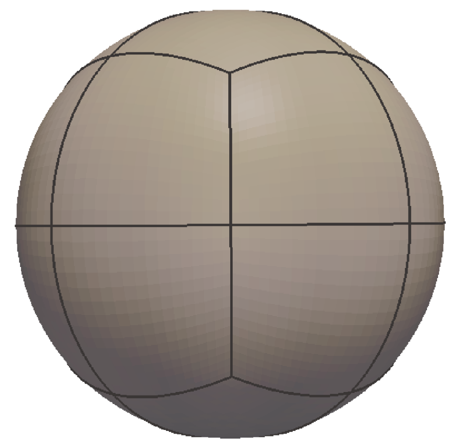
n = 31

Runtime Comparison on Intel i7 3.0 Ghz single core processor





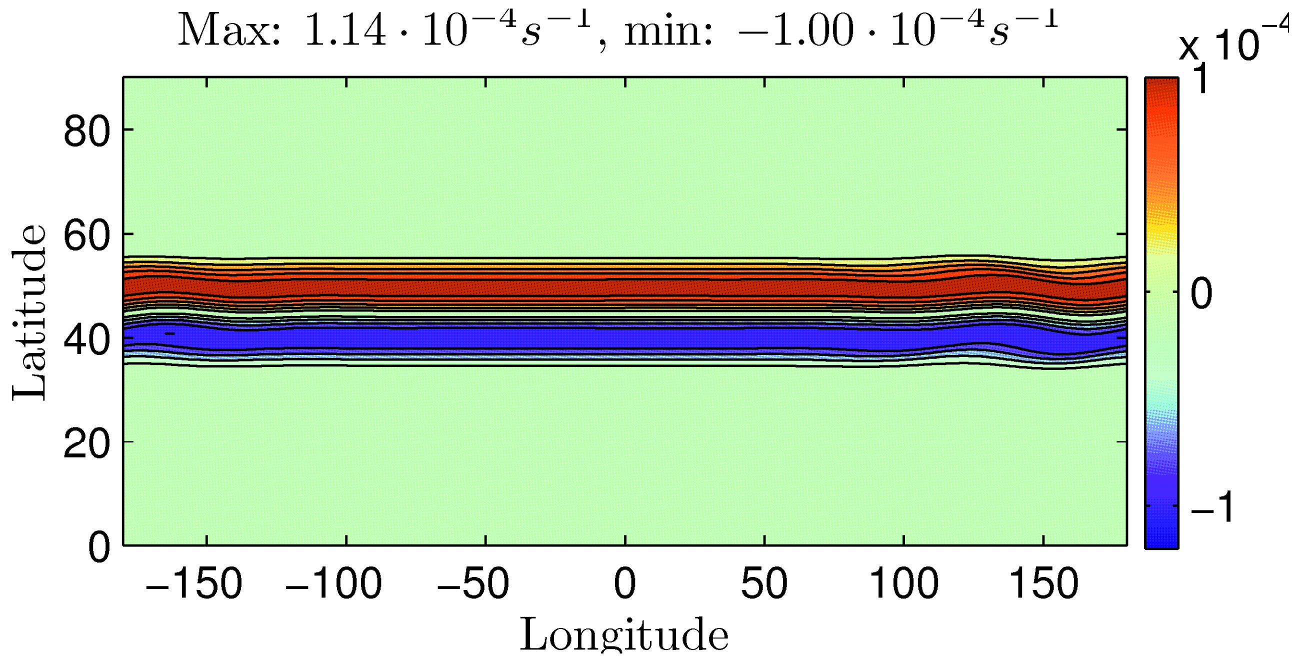
R0



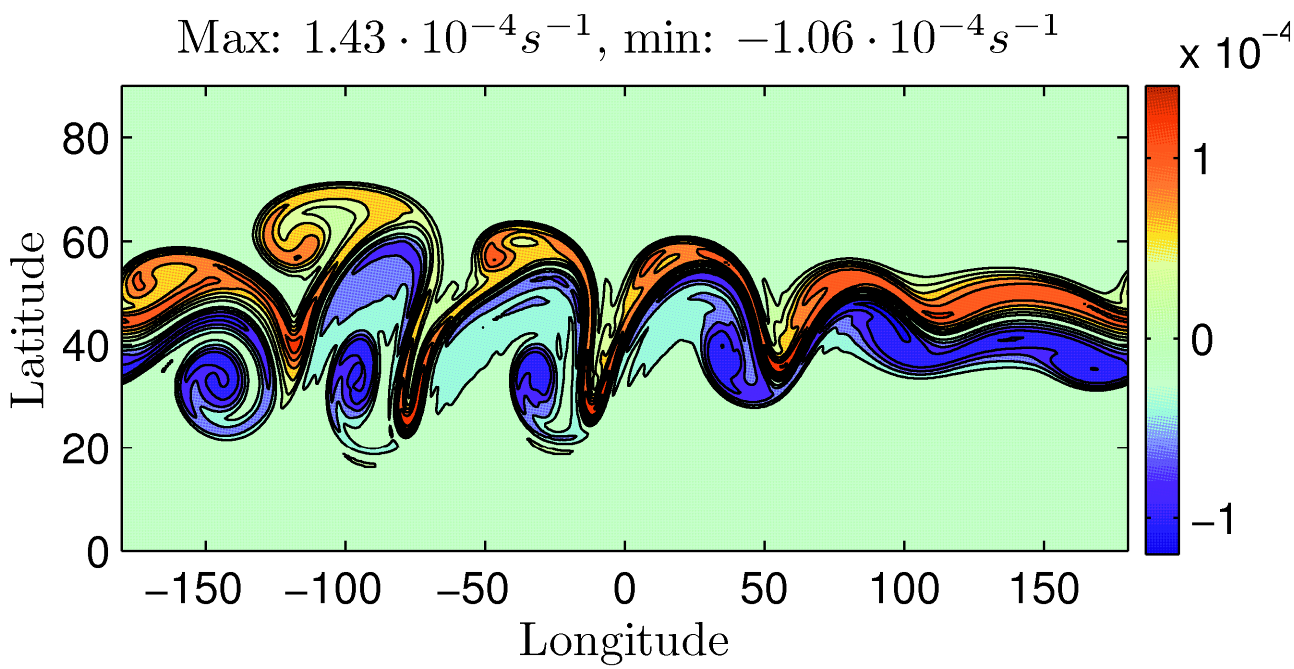
R1

Rapid Evolution of a Highly Unstable Wave in a Mid-Latitude Jet

Rapid cascade of energy from large to small scales $\Longrightarrow$ sharp vorticity gradients

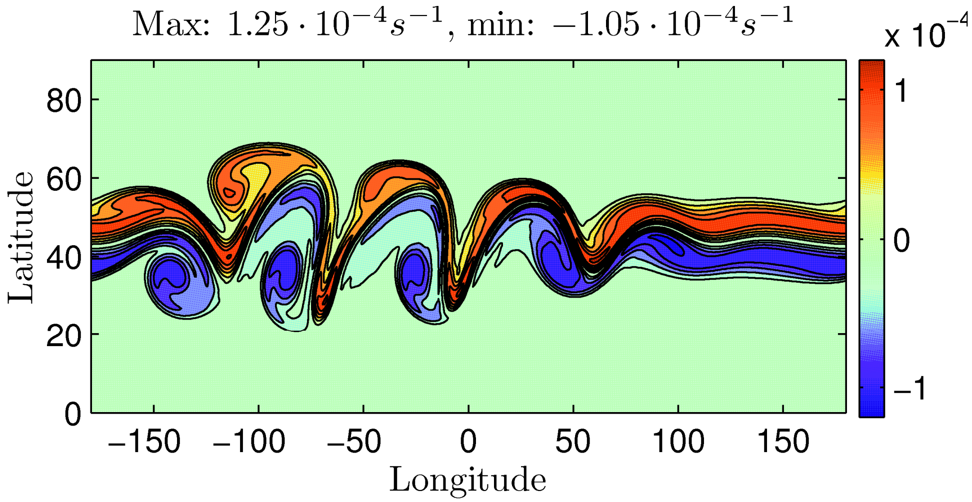


Initial signs of instability by Day 3

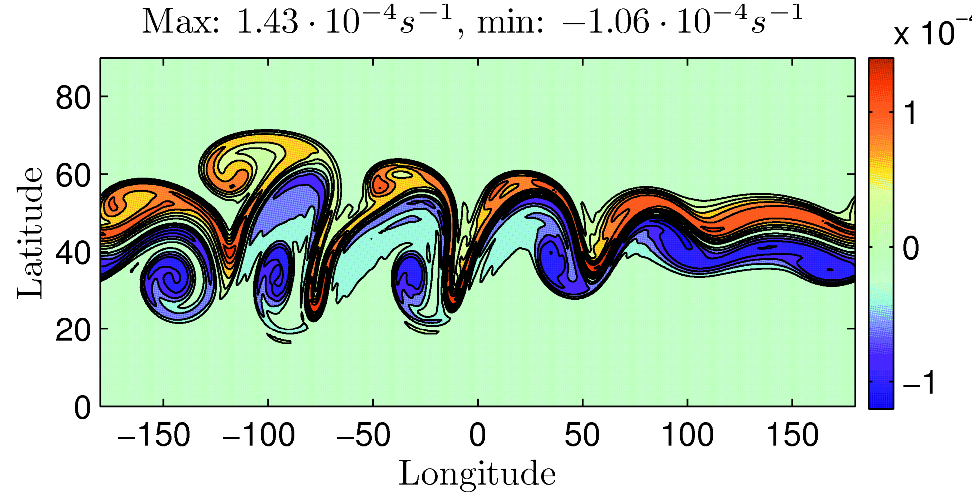


Unstable and tight complex vortical dynamics - Day 6

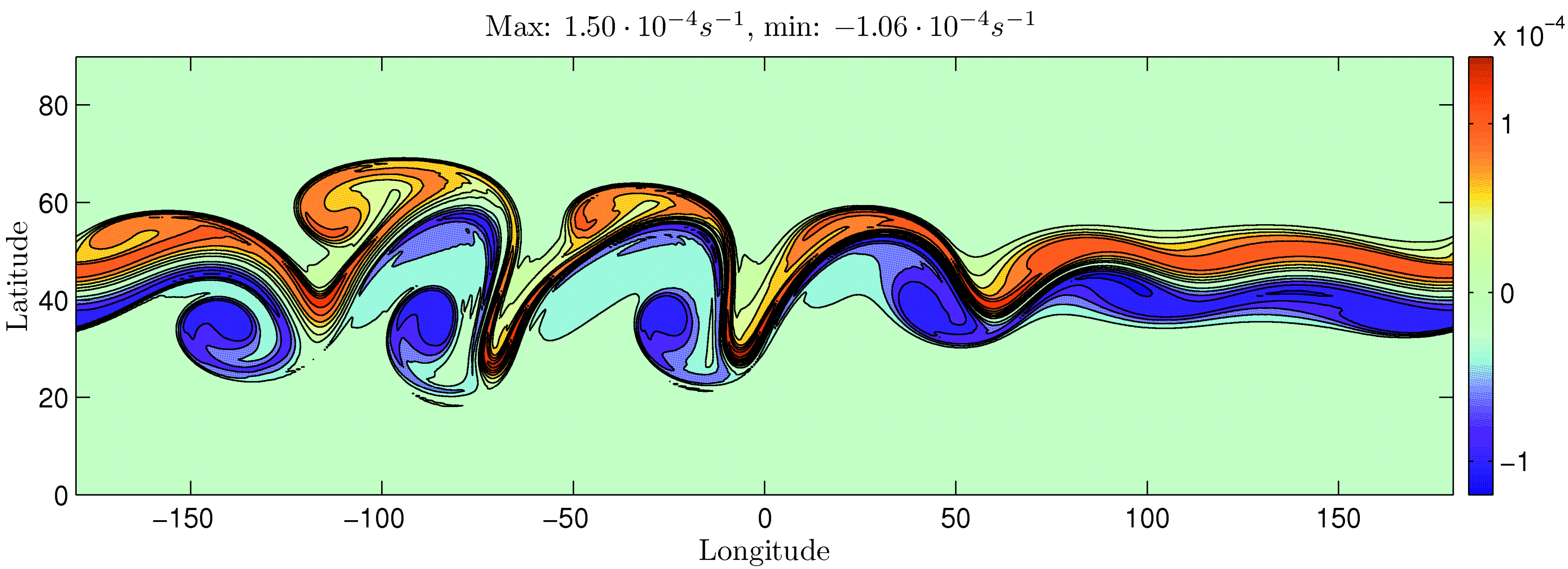
Stencil Size and Hyperviscosity



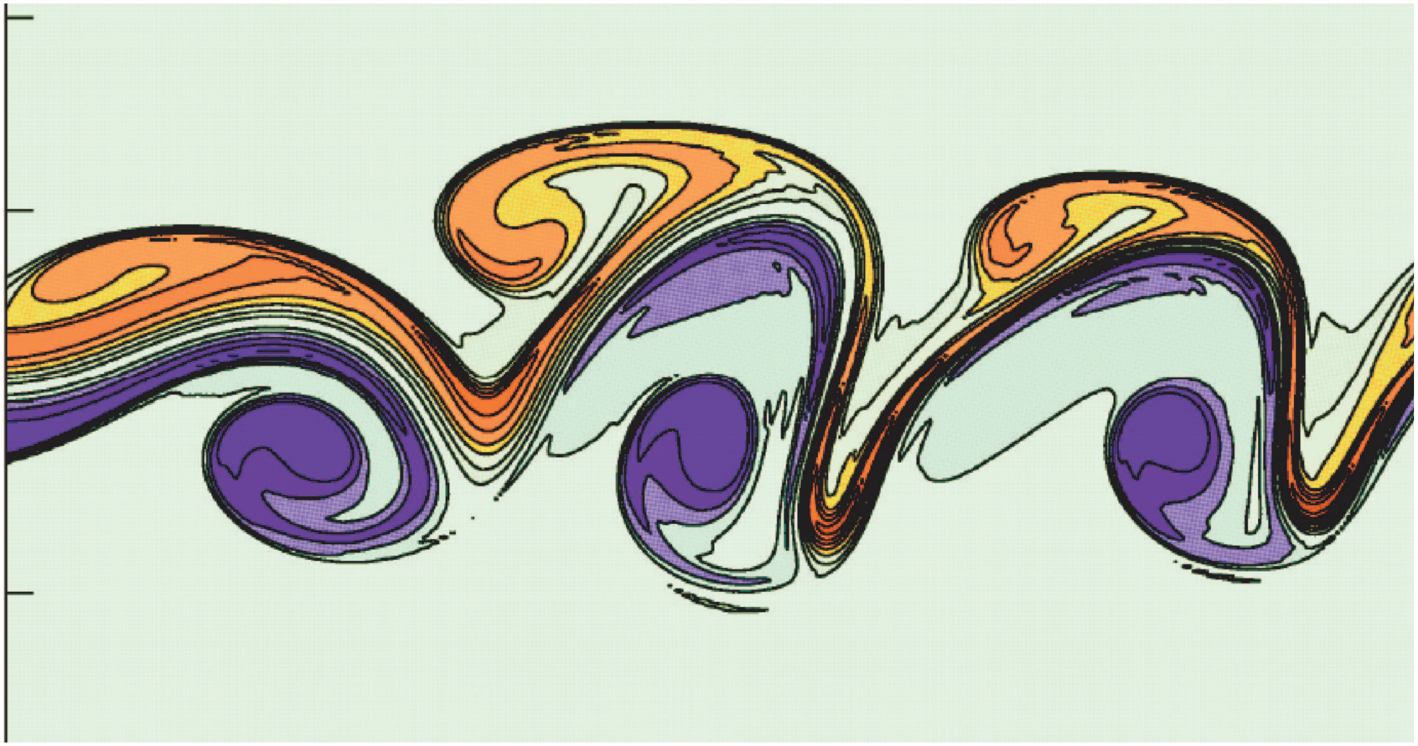
N = 25,600 , n = 31, Δ^4- type



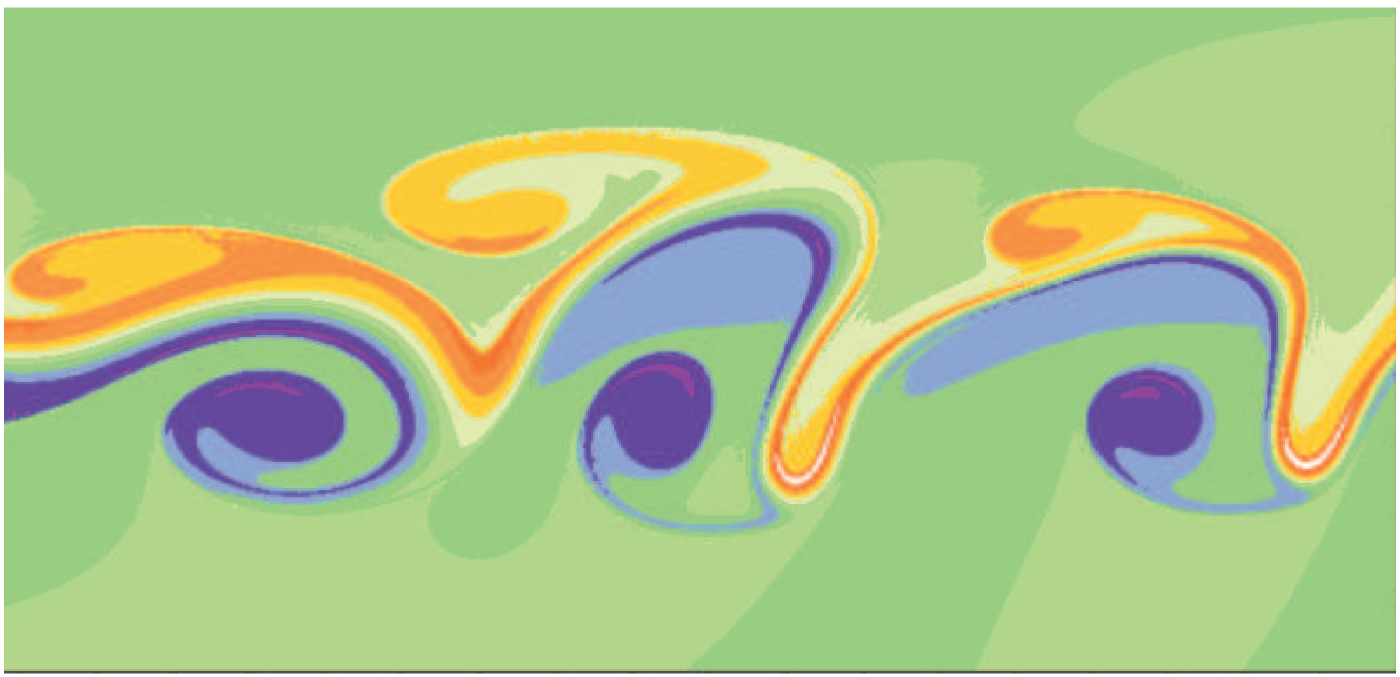
N = 25,600 , n = 101, Δ^{10}- type



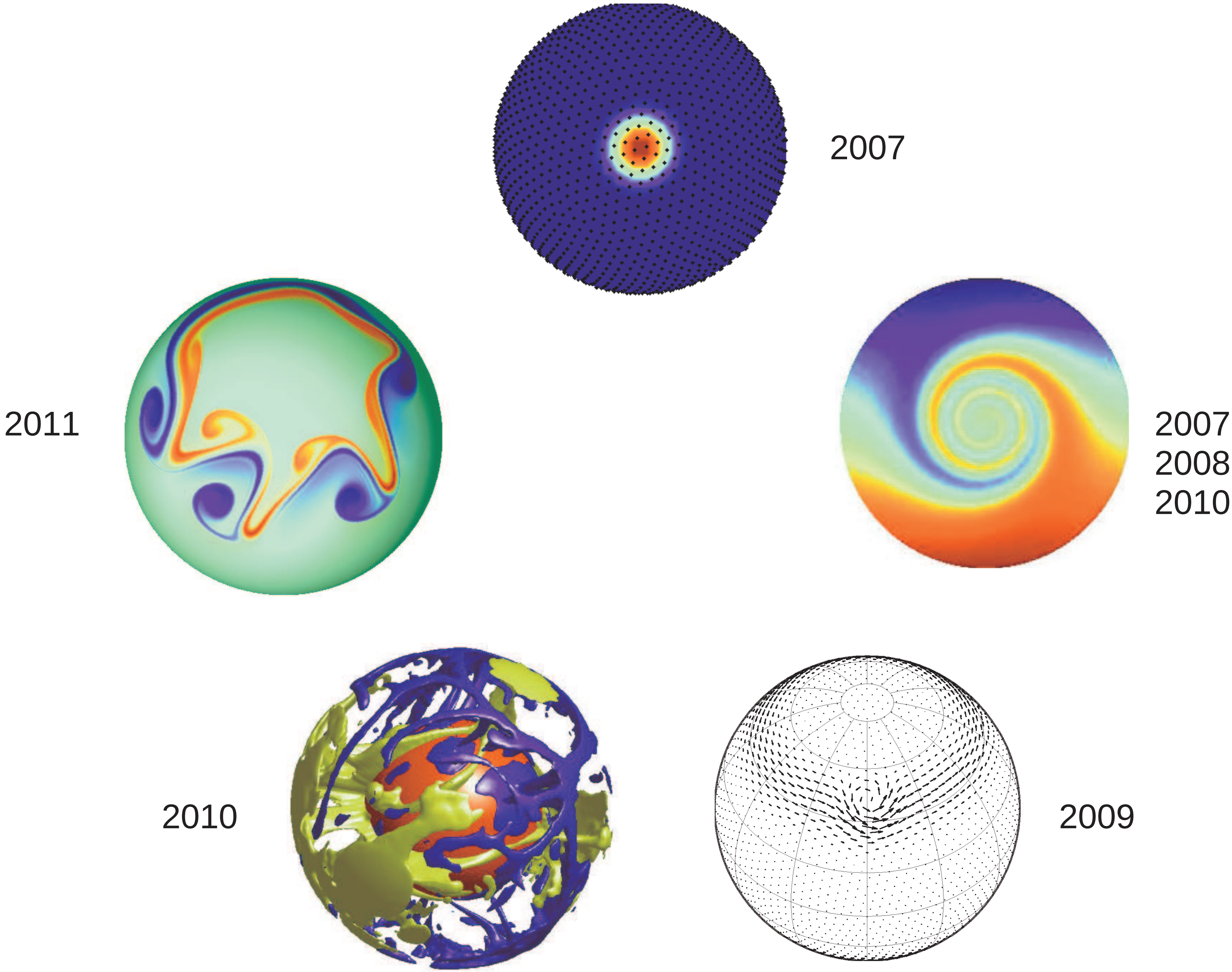
N = 163,842 , n = 31, Δ^4- type



RBF



SEM (St-Cyr et al.)


2007
2011
2007
2008
2010
2010
2009

THANK YOU

Special Thanks to: Prof. Grady B. Wright and Mr. Erik Lehto (Uppsala U.)